MAT 104 Assignment

1 $y = \frac{2\cos 3x}{x^3}$

Using Quotient Rule,

$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\Rightarrow u = 2\cos 3x \; ; \; \frac{du}{dx} = -6\sin 3x$

$v = x^3 \; ; \; \frac{dv}{dx} = 3x$

$\Rightarrow \frac{dy}{dx} = \frac{x^3 \cdot (-6\sin 3x) - (2\cos 3x) \cdot 3x^2}{(x^3)^2}$

$= \frac{-6x^3 \sin 3x - 6x^2 \cos 3x}{x^6}$

2 If $y = xe^{2x}$, Show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

$\Rightarrow \frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 4y$

Using Product Rule, $y = uv$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$\Rightarrow u = x \; , \; \frac{du}{dx} = 1$

$v = e^{2x} \; , \; \frac{dv}{dx} = 2e^{2x}$

$\frac{dy}{dx} = 2e^{2x} \cdot x + e^{2x} \cdot 1$

$= 2xe^{2x} + e^{2x}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$u = 2x \; ; \; \frac{du}{dx} = 2$

$v = e^{2x} \quad \frac{dv}{dx} = 2e^{2x}$

$\frac{d}{dx}(2xe^{2x}) = 2x \cdot 2e^{2x} + e^{2x} \cdot 2 + 2e^{2x}$

$= 4xe^{2x} + 2e^{2x} + 2e^{2x}$

$$\Rightarrow \frac{d^2y}{dx^2} = 4xe^{2x} + 4e^{2x}$$

Recall,

$$\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 4y$$

$$\Rightarrow 4xe^{2x} + 4e^{2x} = 4(2xe^{2x} + e^{2x}) - 4(xe^{2x})$$

$$\Rightarrow 4xe^{2x} + 4e^{2x} = 8xe^{2x} + 4e^{2x} - 4xe^{2x}$$

$$4xe^{2x} + 4e^{2x} = 8xe^{2x} - 4xe^{2x} + 4e^{2x}$$

$$\Rightarrow 4xe^{2x} + 4e^{2x} = 4xe^{2x} + 4e^{2x}$$

$$\therefore \frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 4y \Rightarrow \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

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4 $\int e^x \sin 2x \, dx$

$\Rightarrow$ Integration by Parts

Let $I = \int e^x \sin 2x \, dx$

$\int u \, dv = uv - \int v \, du$

$u = \sin 2x \quad \frac{du}{dx} = 2\cos 2x \Rightarrow du = 2\cos 2x \, dx$

$dv = e^x \quad v = e^x$

$\Rightarrow e^x . \sin 2x - \int e^x . 2\cos 2x \, dx$

$= e^x \sin 2x - \int 2e^x \cos 2x \, dx$

$= e^x \sin 2x - 2\int e^x \cos 2x \, dx$

$\int e^x \cos 2x \, dx$

$u = \cos 2x \quad \frac{du}{dx} = -2\sin 2x \Rightarrow du = -2\sin 2x \, dx$

$dv = e^x \quad v = e^x$

$$\Rightarrow e^x \cdot \cos 2x - \int e^x \cdot (-2\sin 2x)\,dx$$
$$= e^x \cos 2x - \int -2e^x \sin 2x$$
$$= e^x \cos 2x + 2\int e^x \sin 2x\,dx$$

Recall, $I = \int e^x \sin 2x\,dx$

$$\Rightarrow \int e^x \sin 2x\,dx = e^x \sin 2x - 2\left[e^x \cos 2x + 4\int e^x \sin 2x\,dx\right]$$

$$\int e^x \sin 2x\,dx = e^x \sin 2x - 2e^x \cos 2x - 4\int e^x \sin 2x\,dx$$

$$I = e^x \sin 2x - 2e^x \cos 2x - 4I$$
$$I + 4I = e^x \sin 2x - 2e^x \cos 2x$$
$$5I = e^x \sin 2x - 2e^x \cos 2x$$
$$I = \frac{e^x \sin 2x - 2e^x \cos 2x}{5}$$

$$I = \frac{1}{5}\left[e^x \sin 2x - 2e^x \cos 2x\right]$$

$$\Rightarrow \int e^x \sin 2x\,dx = \frac{1}{5}\left[e^x \sin 2x - 2e^x \cos 2x\right] + C$$

OR

$$\int e^x \sin 2x\,dx = \frac{1}{5}e^x(\sin 2x - 2\cos 2x) + C$$

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