

$$A = \begin{vmatrix} 1 & 5 & 2 \\ 1 & 0 & 6 \\ 9 & 1 & 1 \end{vmatrix}$$

$$B = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$C = \begin{vmatrix} 1 & 6 & 2 \\ 4 & 2 & 1 \\ 1 & 7 & 1 \end{vmatrix}$$

Question 1

1) Let  $x = (a, b, c) \Rightarrow x = (5, 4, 3)$

$\therefore Tx = Ax$

$$\begin{vmatrix} 1 & 5 & 2 \\ 1 & 0 & 6 \\ 9 & 1 & 1 \end{vmatrix} \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$$

$$= 5 \begin{bmatrix} 1 \\ 1 \\ 9 \end{bmatrix} + 4 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 5 \\ 45 \end{bmatrix} + \begin{bmatrix} 20 \\ 0 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ 18 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 31 \\ 23 \\ 50 \end{bmatrix}$$

2)

$B+C$

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 6 & 2 \\ 4 & 2 & 1 \\ 1 & 7 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 7 & 4 \\ 5 & 3 & 3 \\ 3 & 9 & 2 \end{vmatrix}$$

$$\det(B+C) = 2(-2) - 7(1) + 4(36)$$

$$= -4 - 7 + 144$$

$$= 93 \therefore \text{It is not } 0$$

$(B+C)$  is not equal to 0; Hence the rank is 3

3)

$$A = \begin{vmatrix} 1 & 5 & 2 \\ 1 & 0 & 6 \\ 9 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} 0 & 6 \\ 1 & 1 \end{vmatrix} - 5 \begin{vmatrix} 1 & 6 \\ 9 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 0 \\ 9 & 1 \end{vmatrix}$$

$$\Rightarrow 1(0-6) - 5(1-54) + 2(1-9)$$

$$= -6 + 265 + 2$$

$$= 261$$

$\therefore$  Matrix A is a non-singular matrix.

$$B = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}$$

$$= 1(1-4) - 1(1-4) + 2(2-2)$$

$$= 1(-3) - 1(-3) + 2(0)$$

$$= 0$$

Matrix B is a singular matrix.

$$C = \begin{vmatrix} 1 & 6 & 2 \\ 4 & 2 & 1 \\ 1 & 7 & 1 \end{vmatrix} = 1 \begin{vmatrix} 2 & 1 \\ 7 & 1 \end{vmatrix} - 6 \begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 4 & 2 \\ 1 & 7 \end{vmatrix}$$

$$= 1(2-7) + 6(4-1) + 2(28-2)$$

$$= 65$$

Matrix C is a singular matrix.