19/SCI01/072 - Ogoudu Nkechi Martina
Math 102.

1. $M = p\bar{i} - 6j - 3k$, $N = 4i + 3j - k$, $O = i - 3j + 2k$
Finding the value of P @ M and N perpendicular.
Solution

$$M \cdot N = (P)(4) + (-6)(3) + (-3)(-1)$$
$$= 4P + (-18) + 3$$
$$4P - 18 + 3$$
$$= 4P - 15, \quad 4P = 15, \quad P = 15/4$$

M and N and O are coplanar; Solution
$\bar{M} \cdot (\bar{N} \times \bar{O})$.

$$(\bar{N} \times \bar{O}) = \begin{vmatrix} i & j & k \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = i(6-3) - j(8 - -1) + k(-12-3).$$
$$3i - 9j - 15k.$$

$$\bar{M} \cdot (\bar{N} \times \bar{O})$$
$$(P)(3) + (-6)(-9) + (-3)(-15).$$
$$3p + 54 + 45, \quad 3p + 99 = 0.$$
$$3p = -99$$
$$p = -99/3$$
$$P = -33$$

Q. Find the direction cosines and the unit vector along the sum of $3i + 2j + 5k$, $2i - j + 6k$ and $5i + 2j - 3k$.
Solution.

Sum of vectors $= (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$.
$$= 10i + 3j + 8k.$$

Direction cosines of $10i + 3j + 8k$.
$$\sqrt{10^2 + 3^2 + 8^2}$$
$$= \sqrt{173}$$
$$= 13.15$$

$$\cos\theta = \frac{10}{|13|} \quad \frac{10}{|13.15|} = 0.76$$

$\cos\theta = 0.76$

$\theta = \cos^{-1}(0.76)$.

$\theta = 40.50°$

$\cos\beta = \dfrac{3}{|13.15|}$ 0.23

$\beta = \cos^{-1}(0.23) = 76.82°$

$\cos\gamma = \dfrac{8}{|13.15|} = 0.61$

$\gamma = \cos^{-1}(0.61)$

$\gamma = 52.55°$

Unit vector $= \dfrac{10i + 3j + 8k}{13.15}$

(3) If $F = 3ui + u^2j + (u+2)k$ and $V = 2ui - 3uj + (u-2k)$, evaluate the integral of $(F \times V)\,du$ from 0 to 1

Solution $(F \times V)$ $\begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix} = i(u^2(u-2) - j(3u(u-2) - 2u(u+2)) + k(3u(u-2)$ $k(3u(-3u) - 2u^3)$

$= \cancel{i(u^3 - 2u^2) - 2u^3 + 4u) - j(}$

$= i[u^3 - 2u^2 - (-3u^2 - 6u)] - j[(3u^2 - 6u) - (2u^2 + 4u)] +$
$k(-9u^2 - 2u^3$

$= i(u^3 - 2u^2 + 3u^2 + 6u) - j(3u^2 - 6u - 2u^2 - 4u) + k(-9u^2 - 2u^3$

$(6u + u^2 + u^3)i - (u^2 - 10u)j + (-2u^3 - 9u^2)k.$

$\int_0^1 (u^3 + u^2 + 6u)i - (u^2 - 10u)j + (-2u^3 - 9u^2)k \cdot du$

$= \int_0^1 \dfrac{u^4}{4} + \dfrac{u^3}{3} + \dfrac{6u^2}{2}\Big)i - \Big(\dfrac{u^3}{3} - \dfrac{10u^2}{2}\Big)j + \Big(-\dfrac{2u^4}{4} - \dfrac{9u^3}{3}\Big)k$

$\left|\Big(\dfrac{u^4}{4} + \dfrac{u^3}{3} + 3u^2\Big)i - \Big(\dfrac{u^3}{3}\Big) - 5u^2\Big)j + \Big(\dfrac{-u^4}{2} - 3u^3\Big)\right|_0^1$

$= \Big|\dfrac{1}{4} + \dfrac{1}{3} + 3\Big)i - \Big[\dfrac{1}{3} - 5\Big]j + \Big(-\dfrac{1}{2} - 3\Big)$

$= {}^{43}/_{12}i + {}^{14}/_{3}j - {}^{7}/_{2}k$