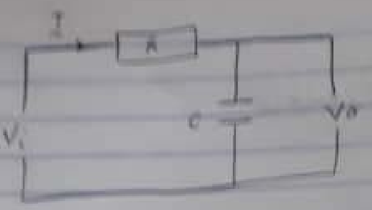
1)



$\left(\frac{V_0}{V_i}\right)(s) = \frac{1}{Ts+1}$ where $T = RC$ $R = 47$ $C = 20\mu F$

$\Rightarrow T = 47 \times 20 \times 10^{-6} = 9.4 \times 10^{-4}$

$G(s) = \frac{1}{Ts+1}$

$G(j\omega) = \frac{1}{9.4 \times 10^{-4} j\omega + 1} \times \frac{9.4 \times 10^{-4} j\omega - 1}{9.4 \times 10^{-4} j\omega - 1}$

$G(j\omega) = \frac{9.4 \times 10^{-4} j\omega - 1}{(9.4 \times 10^{-4})\omega^2 - 1}$

$G(j\omega) = \frac{-1}{(9.4 \times 10^{-4})^2 \omega^2 - 1} + \frac{9.4 \times 10^{-4} j\omega}{(9.4 \times 10^{-4})^2 \omega^2 - 1}$

where $\omega = 2000$ rads^{-1}

$\phi = \tan^{-1}\left[\frac{\frac{9.4 \times 10^{-4}(2000)}{(9.4 \times 10^{-4})\omega^2 - 1}}{\frac{-1}{(9.4 \times 10^{-4})^2 \omega^2 - 1}}\right] = \tan^{-1}\left[\frac{9.4 \times 10^{-4}(2000)}{-1}\right]$

$\phi = -61.99°$

$|G(j\omega)| = \frac{1}{\sqrt{(9.4 \times 10^{-4})^2 \omega^2 + 1^2}} \Rightarrow \frac{1}{\sqrt{(9.4 \times 10^{-4})^2 (2000)^2 + 1^2}} = 0.46$

V_0: amplitude $= 5 \times 0.4696 = 2.348$

$\phi = 2000t - 61.99°$

$V_0 = 2.348 \sin(2000t - 61.99°)$

$V_0 \simeq 2.35 \sin(2000t - 62°)$ V

2) $\frac{x_o}{x_i} = \frac{1}{T^2s^2 + 2\delta Ts + 1}$, $G(s) = \frac{1}{(1 - T^2s^2) + 2\delta Ts}$

$$G(j\omega) = \frac{1}{(1 - T^2\omega^2) + 2\delta T j\omega}$$

$\delta = 0.2$ $T = 0.4$ sec $\omega = 2.5$ rad/s

$$G(j\omega) = \frac{1 - T^2\omega^2 - 2\delta T j\omega}{(1 - T^2\omega^2)^2 + 4\delta^2 T^2\omega^2}$$

$$\Rightarrow \frac{1 - (0.4)^2(2.5)^2 - 2(0.2)(0.4)(2.5)j}{(1 - (0.4)^2(2.5)^2)^2 + 4(0.2)^2(0.4)^2(2.5)^2}$$

$$G(j\omega) = 0 - 2.5j$$

$$\phi = \tan^{-1}\left[\frac{2.5}{0}\right] \Rightarrow \tan^{-1}(\infty) = 90^\circ$$

$$|G_{j\omega}| = \sqrt{0^2 + (2.5^2)}$$

$$= 2.5$$

amplitude $\Rightarrow 6 \times 2.5$

$= 15$

GGS FRANCIS SOIBI
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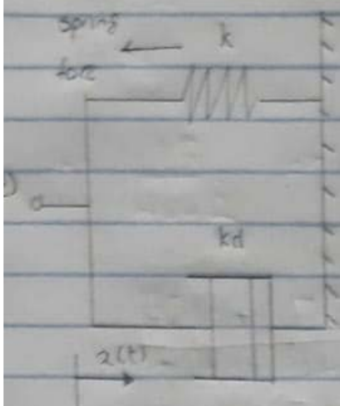


$F_{spring} \Rightarrow k(x-0)$

$F_{damper} \Rightarrow k_d \dfrac{d(x-0)}{dt}$

$F(t) \Rightarrow F(t)$

Newton's law $\Rightarrow$ $F(t) - k(x-0) - k_d \dfrac{d(x-0)}{dt} = 0$

$0 = F(t) - kx - k_d \dfrac{dx}{dt}$ Laplace transform $\Rightarrow$ $F(s) - k\,x(s) - k_d s\,x(s)$

$$F(s) - k\,x(s) - k_d s\,x(s) = 0$$

$$F(s) = [k + k_d s]\,x(s)$$

$$G(s) = \frac{x(s)}{F(s)} = \frac{1}{k + k_d s} \Rightarrow \frac{1/k}{1 + [k_d/k]s} \quad \text{compare to} \quad \frac{1/k}{Ts+1}$$

$$\Rightarrow T = \frac{k_d}{k} \Rightarrow \frac{0.03}{4\times10^{3}} = 0.75\times10^{-5} \text{ seconds} \Rightarrow 7.5\times10^{-6}\,s$$

$7.5\mu s$

$$\frac{\Delta\theta_1(t)}{\Delta\theta_2(t)} = \frac{1}{T}\left(e^{-t/T}\right)$$

$$\frac{\theta - \theta_2}{\theta_2 - \theta_1} = \frac{1}{T}\left(e^{-t/T}\right)$$

$$\theta - \theta_2 = (\theta_2 - \theta_1)\frac{\left(e^{-t/T}\right)}{T}$$

$$\theta = \theta_2 +$$

5 i) $\frac{2}{0.2s+0.5} \Rightarrow \frac{2/0.5}{0.2s/0.5 + 1}$

$\Rightarrow \frac{4}{0.4s+1}$ compare to $\frac{K}{Ts+1}$

4 = D.C gain   0.4 = Time constant

ii) $\frac{0.2}{0.05s+0.1} = \frac{0.2/0.1}{0.05s/0.1 + 0.1/0.1} = \frac{2}{0.5s+1}$

2 = D.C gain   0.5 = Time constant

iii) $\frac{2}{3s+1} \Rightarrow$ 2 = D.C gain, 3 = Time constant

iv) $\frac{16}{8s+4} \Rightarrow \frac{16/4}{8/4 s + 1} = \frac{4}{2s+1}$

4 = D.C gain   2 = Time constant

6) $\frac{\omega}{\theta}(s) = \frac{K_m}{T_m s + 2}$

$K_m = 15 s^{-1}$

$T_m = 4$

$\Rightarrow \frac{15}{4s+2} \Rightarrow \frac{15/2}{4s/2 + 1} = \frac{7.5}{2s+1}$

DC gain $= 7.5 s^{-1}$

time constant $= 2$

4) $\theta_i(t) = ct$

$$\theta_i(s) = \frac{c}{s^2}$$

$$\frac{\theta_o(s)}{\theta_i} = \frac{1}{3s+1}$$

$$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$$

$$\theta_o(s) = \frac{c}{s^2(3s+1)}$$

$$\theta_o(t) = \frac{c/3}{s^2(s+1/3)}$$

$$\theta_o(t) = c\left[t - 3(1 - e^{-t/3})\right] \quad \text{— (i)}$$

~~at ∞~~ when t is large

~~$\theta_o(t)$~~ $\theta_o(t) = c[t - 3(1)]$

$\theta_o(t) = ct - 3c$

$\theta_e = \theta_i - \theta_o = ct - (ct - 3c) = 3c$

$T = 3 \quad c = 4\,mm/s$

after 2 seconds

$\theta_i = 4 \times 2 = 8\,mm$

$\theta_e = c \times 3 \Rightarrow 4\,mm/s \times 3 = 12\,mm$ (at steady s

θ_o from (i)

$\theta_o = 4[2 - 3(1 - e^{-2/3})] = 2.161\,mm$

3) 63.2% 99.9%

k_1 $t = T$ $t = 4T$

$k_2 \cdot x$

$\delta = k_3$

$$\frac{\omega}{k_m x} = \frac{1}{Ts+1}$$

$$T = \frac{1}{k_3} \qquad k_m = \frac{k_1 k_2}{k_3}$$

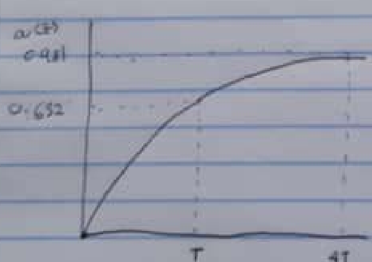
$$\omega = \frac{k_m x}{Ts+1}$$

taking the laplace transform of step input

$$\omega = \frac{k_m x}{s}\left(\frac{1}{Ts+1}\right)$$

$$\frac{k_m x}{s}\left(\frac{1/T}{s+1/T}\right)$$

$$\omega(t) \Rightarrow k_m x\left[1 - e^{-t/T}\right]$$



at $t = 0$ $k_m x\{1 - e^{0}\} = 0(k_m x)$

at $t = T$ $k_m x\{1 - e^{-T/T}\} = 0.632 k_m x$

$\Rightarrow 0.632\, k_m x$

at $t = 4T$ $k_m x\{1 - e^{-4T/T}\} =$

$0.981\, k_m x$

for $t = T$

$\Delta\% = [0.632 - 0]\times 100\% = \underline{63.2\%}$

for $t = 4T$

$\Delta\% = [0.981 - 0]\times 100\% = \underline{98.1\%}$

2) E_2 = new energy
E_1 = inital energy

$E_2 = mc\Delta\theta \Rightarrow E_2 = mc(\theta - \theta_1)$
$E_1 = mc\Delta\theta \Rightarrow E_1 = mc(\theta_2 - \theta_1)$
where θ is the new temp of the metal

$$G(s) = \frac{E_2}{E_1} = \frac{\cancel{mc}(\theta - \theta_1)}{\cancel{mc}(\theta_2 - \theta_1)} = \frac{1}{Ts+1}$$

$$\Rightarrow \frac{\theta - \theta_1}{\theta_2 - \theta_1}(s) = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

let $\theta_2 - \theta_1(s) = k(t)$ — (1)

$$(\theta - \theta_1)(s) = \frac{k(t)}{Ts+1}$$

$$(\theta - \theta)(s) = \frac{k(t)(1/T)}{s + 1/T}$$

taking the laplace transform of $k(t)$

$$= \frac{k}{s}$$

$$(\theta - \theta_1)(s) = \frac{k(1/T)}{s(s + 1/T)}$$

inverse laplace transform

$$(\theta - \theta_1) = k\left[1 - e^{-t/T}\right]$$

from (1) $k = \theta_2 - \theta_1$

$$(\theta - \theta_1) = (\theta_2 - \theta_1)\left[1 - e^{-t/T}\right]$$

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