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Math 104 Assignment

1.) $y = \frac{2\cos 3x}{x^3}$, find $\frac{dy}{dx}$

Let $u = 2\cos 3x$; $du/dx = -6\sin 3x$

$v = x^3$; $dv/dx = 3x^2$

Using quotient rule; $\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

$$\frac{dy}{dx} = \frac{(x^3)(-6\sin 3x) - (2\cos 3x)(3x^2)}{(x^3)^2} = \frac{-6x^3\sin 3x - 6x^2\cos 3x}{x^6}$$

$$= \frac{-6x^2(x\sin 3x + \cos 3x)}{x^{\cancel{6}4}} = \frac{-6(x\sin 3x + \cos 3x)}{x^4}$$

2.) Prove $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ $y = xe^{2x}$

$$\frac{dy}{dx} = \frac{d(x)}{dx} \times e^{2x} + \frac{d}{dx}(e^{2x}) \times x$$

$$= (1e^{2x}) + (2e^{2x} \times x) = e^{2x} + 2xe^{2x}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(e^{2x}) + \left[\frac{d}{dx}(2x) \times e^{2x} + \frac{d}{dx}(e^{2x}) \times 2x\right]$$

$$= 2e^{2x} + (2xe^{2x}) + (2e^{2x} \times 2x) = 2e^{2x} + 2e^{2x} + 4xe^{2x}$$

$$\frac{d^2y}{dx^2} = 4e^{2x} + 4xe^{2x}$$

$$\therefore \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

$$\Rightarrow 4e^{2x} + 4xe^{2x} - 4(e^{2x} + 2xe^{2x}) + 4(xe^{2x})$$

$$4e^{2x} + 4xe^{2x} - 4e^{2x} - 8xe^{2x} + 4xe^{2x}$$

$$4xe^{2x} - 8xe^{2x} + 4xe^{2x} + 4e^{2x} - 4e^{2x} = 0$$

$$0 + 0 = 0$$

$$0 = 0 \quad Q.E.D$$

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4.) $\int e^x \sin 2x \; dx$

$$= \int \sin 2x \times e^x \, dx$$

$$= \sin 2x \times e^x - \int e^x \times \cos 2x \times 2 \, dx$$

$$= \sin 2x \times e^x - 2 \times \int e^x \times \cos 2x \, dx$$

$$= \sin 2x \times e^x - 2 \times \int \cos 2x \times e^x \, dx$$

$\int u\,dv = uv - \int v\,du$ where $u = \cos 2x$, $dv = e^x dx$

$$= \sin(2x) \times e^x - 2\left(\cos 2x \times e^x - \int e^x \times (-\sin 2x) \times 2 \, dx\right)$$

$$= \sin 2x \times e^x - 2\left(\cos 2x \times e^x - 1 \times (-2) \times \int e^x \times \sin 2x \, dx\right)$$

$$= \sin 2x \times e^x - 2\left[\cos 2x \times e^x + 2x \int e^x \times \sin 2x \, dx\right)$$

$$= \int e^x \times \sin 2x \, dx = \sin 2x \times e^x - 2\left(\cos 2x \times e^x + 2x \int e^x \times \sin 2x \, dx\right)$$

$$= \int e^x \times \sin 2x \, dx = \sin 2x \times e^x - 2\cos 2x \times e^x - 4x \int e^x \times \sin 2x \, dx$$

$$= \int e^x \times \sin 2x \, dx + 4x \int e^x \times \sin(2x) \, dx = \sin 2x \times e^x - 2\cos 2x \times e^x$$

$$= 5 \times \int e^x \times \sin 2x \, dx = \sin 2x \times e^x - 2\cos 2x \times e^x$$

$$= \int e^x \times \sin 2x \, dx = \frac{\sin 2x \times e^x}{5} - \frac{2\cos(2x) \times e^x}{5}$$

$$= \int e^x \times \sin 2x \, dx = \frac{\sin 2x \times e^x - 2e^x \times \cos 2x}{5}$$

$$\therefore \int e^x \sin 2x \, dx = \frac{\sin 2x \times e^x - 2e^x \times \cos 2x}{5} + c$$