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DEPARTMENT: COMPUTER SCIENCE

MATRIC NO: 19/SCI01/015

ASSIGNMENT

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ASSIGNMENT

1. $\dfrac{(3x-1)}{(x-1)(x-2)(x-3)}\,dx$

Solution

$$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)}\,dx$$

Resolve

$$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

Find the L.C.M of the right hand side

$$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

Let $x = 1$

$$3(1)-1 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$$

$$2 = A(-1)(-2) + B(0)(-2) + C(0)(-1)$$

$$\frac{\cancel{2}}{\cancel{2}} = \frac{\cancel{2}A}{\cancel{2}}$$

$A = 1$

Let $x = 2$

$$3(2)-1 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$$

$$5 = A(0)(-1) + B(1)(-1) + C(-1)(0)$$

$$\frac{5}{-1} = \frac{\cancel{-1}B}{\cancel{-1}}$$

$B = -5$

Let $x = 3$

$$3(3)-1 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$$

$$8 = A(1)(0) + B(2)(0) + C(2)(1)$$

$\frac{8}{2} = \frac{\cancel{2}C}{\cancel{2}}$

$C = 4$

$\therefore$ The resolve is $\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$

$$\Rightarrow \int \frac{(3x-1)}{(x-1)(x-2)(x-3)}\,dx = \int \frac{1}{(x-1)}\,dx - \int \frac{5}{(x-2)}\,dx + \int \frac{4}{(x-3)}\,dx$$

$$= \ln(x-1) - 5\int \frac{1}{(x-2)}\,dx + 4\int \frac{1}{(x-3)}\,dx$$

$$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$$

(2) $\frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$

Solution

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$$

Resolve

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$$

Find the L.C.M of the right hand side

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1) + [(Bx+C)(x+2)]}{(x+2)(x^2+1)}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$x^2+x+1 = A(x^2+1) + [(Bx+C)(x+2)]$

Let $x = -2$

$(-2)^2 + (-2) + 1 = A((-2)^2+1) + [(B(-2)+C)(-2+2)]$.

$4 - 2 + 1 = 5A + 0$

$\frac{3}{5} = \frac{\cancel{5}A}{\cancel{5}}$

$A = \frac{3}{5}$

From the R.H.S

$A(x^2+1)+\boxed{(Bx+C)(x+2)}$

$= Ax^2+A+\boxed{Bx^2+2Bx+Cx+2C}$

$= Ax^2+A+Bx^2+2Bx+Cx+2C$

Collect like terms

$= Ax^2+Bx^2+2Bx+Cx+A+2C$

$= x^2(A+B)+x(2B+C)+A+2C$

$x^2+x+1 = x^2(A+B)+x(2B+C)+A+2C$

Compare the coefficient

$1 = A+B$ ----(i)

$1 = 2B+C$ -----(ii)

$1 = A+2C$ -----(iii)

Substitute $A = 3/5$ in equation (i)

$1 = \frac{3}{5}+B$

$B = 1-\frac{3}{5}$

$B = \frac{2}{5}$

Substitute $B = 2/5$ in equation (ii)

$1 = 2\left(\frac{2}{5}\right)+C$

$1 = \frac{4}{5}+C$

$C = 1-\frac{4}{5}$

$C = \frac{1}{5}$

∴ The resolve is $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$

$$\Rightarrow \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{3}{5(x+2)}\,dx + \int \frac{2x+1}{5(x^2+1)}\,dx$$

$$= \int \frac{3}{5(x+2)}\,dx + \frac{1}{5}\int \frac{2x+1}{(x^2+1)}\,dx$$

$$= \frac{3}{5}\int \frac{1}{(x+2)}\,dx + \frac{1}{5}\left[\int \frac{2x}{(x^2+1)}\,dx + \int \frac{1}{(x^2+1)}\,dx\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\left[\frac{\cancel{2x}}{u} \times \frac{1}{\cancel{2x}}\,du + \arctan(x)\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\left[\frac{1}{u}\,du + \arctan(x)\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\left[\ln(x^2+1) + \arctan(x)\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \frac{\arctan(x)}{5} + C$$

$$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \frac{\arctan(x)}{5} + C$$

(3) $\dfrac{(x^2+1)}{(x-3)(x-2)^2}\,dx$

Solution

$$\int \frac{(x^2+1)}{(x-3)(x-2)^2}\,dx$$

Resolve

$$\frac{(x^2+1)}{(x-3)(x-2)^2} = \frac{(x^2+1)}{(x-3)(x^2-4x+4)}$$

$$\frac{(x^2+1)}{(x-3)(x^2-4x+4)} = \frac{A}{(x-3)} + \frac{Bx+C}{(x^2-4x+4)}$$

Find the L.C.M of the right hand side

$$\frac{(x^2+1)}{(x-3)(x^2-4x+4)} = \frac{A(x^2-4x+4)+\left[(Bx+C)(x-3)\right]}{(x-3)(x^2-4x+4)}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$(x^2+1) = A(x^2-4x+4) + [(Bx+C)(x-3)]$

Let $x=3$

$(3)^2+1 = A((3)^2-4(3)+4) + [(B(3)+C)(3-3)]$

$10 = A(1)$

$A = 10$

From the R.H.S

$A(x^2-4x+4) + [(Bx+C)(x-3)]$

$= Ax^2-4Ax+4A + [Bx^2-3Bx+Cx-3C]$

$= Ax^2-4Ax+4A+Bx^2-3Bx+Cx-3C$

collect like terms

$= Ax^2+Bx^2-4Ax-3Bx+Cx+4A-3C$

$= x^2(A+B)+x(-4A-3B+C)+4A-3C$

$(x^2+1) = x^2(A+B)+x(-4A-3B+C)+4A-3C$

compare the coefficient

$1 = A+B$ ----- (i)

$0 = -4A-3B+C$ ------ (ii)

$1 = 4A-3C$ ------ (iii)

substitute $A=10$ in equation (i)

$1 = A+B$

$1 = 10+B$

$B = 1-10$

$B = -9$

substitute $B=-9$ and $A=10$ in equation (ii)

$0 = -4A-3B+C$

$0 = -4(10)-3(-9)+C$

$0 = -40+27+C$

$0 = -13+C$

$C = 13$

$\therefore$ The resolve is $\dfrac{(x^2+1)}{(x-3)(x-2)^2} = \dfrac{10}{(x-3)} - \dfrac{9x+13}{(x^2-4x+4)}$

$$\Rightarrow \int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{10}{(x-3)}\,dx - \int \frac{9x+13}{(x^2-4x+14)}\,dx$$

$$= 10\int \frac{1}{(x-3)}\,dx - \int \frac{9x}{(x^2-4x+4)}\,dx + \int \frac{13}{(x^2-4x+4)}\,dx$$

$$= 10\ln(x-3) - 9\ln(x-2) + \frac{18}{(x-2)} - \frac{13}{(x-2)}$$

$$= 10\ln(x-3) - 9\ln(x-2) + \frac{18-13}{(x-2)}$$

$$= 10\ln(x-3) - 9\ln(x-2) + \frac{5}{(x-2)} + C$$

$$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = 10\ln(x-3) - 9\ln(x-2) + \frac{5}{(x-2)} + C$$

4) $\dfrac{(x^3+x^2+x+1)}{(x-1)}\,dx$

Solution

$$\int \frac{(x^3+x^2+x+1)}{(x-1)}\,dx = \int \frac{x^3}{(x-1)}\,dx + \int \frac{x^2}{(x-1)}\,dx + \int \frac{x}{(x-1)}\,dx + \int \frac{1}{(x-1)}\,dx$$

$$= \frac{2x^3+3x^2+6x-11}{6} + \ln(x-1) + \frac{x^2}{2} + x + \ln(x-1) + (x-1)$$
$$+ \ln(x-1) + \ln(x-1) + C$$

$$= \frac{2x^3+3x^2+6x-11}{6} + \frac{x^2}{2} + x + x - 1 + \ln(x-1) + \ln(x-1) +$$
$$\ln(x-1) + \ln(x-1) + C$$

$$= \frac{2x^3+3x^2+6x-11+3x^2+6x+6x-6}{6} + 4\ln(x-1) + C$$

$$= \frac{2x^3+6x^2+18x-17}{6} + 4\ln(x-1) + C$$

$$\therefore \int \frac{x^3+x^2+x+1}{(x-1)}\,dx = \frac{2x^3+6x^2+18x-17}{6} + 4\ln(x-1) + C$$