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1. $$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)}\,dx$$

Solution

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{x-3)}$$

Find the lcm of the right hand side

$$= \frac{A(x-2)(x-3)\ B(x-1)(x-3)\ C(x-2)(x-1)}{(x-1)(x-2)(x-3}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

let $x=1$

$3(1)-1 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$

$2 = A(-1)(-2) + B(-2) + C(-1)$

$\frac{\cancel{2}}{2} = \frac{\cancel{2}A}{\cancel{2}}$

$A = 1$

let $x=2$

$3(2)-1 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$

$5 = 0 + (-B) + 0$

$\frac{5}{-1} = \frac{-B}{-1}$

$B=-5$

let $x=3$

$3(3)-1 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$

$0 + B(2)0 + C(2)(1)$

$\frac{8}{2} = \frac{2C}{2}$ $C=4$

∴ The resolve is $\frac{3(n-1)}{(n-1)(n-2)(n-3)} = \frac{1}{n-1} - \frac{5}{n-2} + \frac{4}{n-3}$

$\Rightarrow \int \frac{(3n-1)}{(n-1)(n-2)(n-3)} dn = \int \frac{1}{(n-1)} dn - \int \frac{5}{n-2} dn + 4\int \frac{1}{n-3} dn$

$= \ln(n-1) - 5\int \frac{1}{n-2} dn + 4\int \frac{1}{n-3} dn$

$= \ln(n-1) - 5\ln(n-2) + 4\ln(n-3) + c$

(2) $\frac{n^2+n+1}{(n+2)(n^2+1)}$

Solution

$\int \frac{n^2+n+1}{(n+2)(n^3+1)} dn$

Resolve

$\frac{n^2+n+1}{(n+2)(n^2+1)} = \frac{A}{n+2} + \frac{Bn+C}{n^2+1}$

Find the lcm of the right hand side

$\frac{n^2+n+1}{(n+2)(n^2+1)} = A(n^2+1) + [(Bn+C)(n+2)]$

let n be 2

$(-2)^2 + (-2) + 1 = A((-2)^2+1) + [B(-2)+C](-2+2)$

$\frac{3}{5} = \frac{\cancel{5}A}{\cancel{5}}$

$A = 3/5$

From the right hand side

$A(n^2+1) + [B(\cancel{x}+c)(n+2)]$

$= An^2 + A + [Bn^2 + 2Bn + cn + 2c]$

$= An^2 + A + Bn^2 + 2Bn + cn + 2c$

CLT

$= An^2 + Bn^2 + 2Bn + cn + A + 2c$

$= n^2(A+B) + n(2B+c) + A + 2c$

$n^2 + n + 1 = n^2(A+B) + n(2B+c) + A + 2c$

compare the co-efficient

$1 = A + B \quad \text{-----} (i)$

$1 = 2B + C \quad \text{-----} (ii)$

$1 = A + 2C \quad \text{-----} (iii)$

Substitute $A = 3/5$ in (i)

$$1 = \frac{3}{5} + B$$

$$B = 1 - \frac{3}{5}$$

$$B = 2/5$$

Substitute $B = 2/5$ in equation (ii)

$$1 = 2\left(\frac{2}{5}\right) + C$$

$$1 = 4/5 + C$$

$$C = 1 - 4/5$$

$$C = 1/5$$

$\therefore$ The resolve is $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$

$$\Rightarrow \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{3}{5(x+2)}\,dx + \int \frac{2x+1}{5(x^2+1)}\,dx$$

$$= \int \frac{3}{5(x+2}\,dx + \frac{1}{5}\int \frac{2x+1}{x^2+1}\,dx$$

$$= 3/5 \int \ln(x+2 + \frac{1}{5}\left[\frac{\cancel{2x}}{u} + \frac{1}{\cancel{2x}}\,du + \right.$$

$$= 3/5 \ln(x+2) + 1/5\left[\ln(x^2+1) + \text{arctan}\right.$$

$$= 3/5 \ln(x+2) + 1/5 (\ln(x^2+1) + \arctan(x)($$

$$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = 3/5 \ln(x+2) + 1/5 \ln(x^2+1) + \frac{\arctan(x}{5}$$

3 $\dfrac{(x^2+1)}{(x-3)(x-2)^2}\,dx$

Solution

$$\int \frac{(x^2+1)}{(x-3)(x-2)^2}\,dx$$

Resolve

$$\frac{(x^2+1)}{(x-3)(x-2)^2} = \frac{(x^2+1)}{(x-3)(x^2-4x+4}$$

$$\frac{(x^2+1)}{(x-3)(x^2-4x+4)}$$

$$\frac{(x^2+1)}{(x-3)(x^2-4x+4)} = \frac{A}{(x-3)} + \frac{Bx+C}{(x^2-4x+4)}$$

Find the l.c.m of the R.H.S

$$\frac{(x^2+1)}{(x-3)(x^2-4x+4)} = \frac{A(x^2-4x+4+[Bx+C)(x-3)}{(x-3)(x^2-4x+4}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$$(x^2+1) = A(x^2-4x+4)+[Bx+C)(x-3)]$$

let $x = 3$

$$(3)^2+1 = A((3)^2-4(3)+4)+[B(3)+C)(3-$$

$$10 = A(1)$$

$$A = 10$$

From the R.H.S

$$A(x^2-4x+4)+[Bx+C)(x-3]$$

$$= Ax^2 - 4Ax + 4A + [Bx^2 - 3Bx + Cx - 3C$$

$$= Ax^2 - 4Ax + 4A + Bx^2 - 3Bx + (x-$$

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$$= Ax^2 + Bx^2 - 4Ax - 3Bx + Cx + 4A$$

$$= x^2(A+B) + x(-4A - 3B + C) + 4$$

Compare the coefficient

$$1 = A + B \quad -(i)$$

$$0 = -4A - 3B + C \quad -(ii)$$

$$1 = 4A - 3C \quad -(iii)$$

Substitute $A = 10$ in equation (i)

$$1 = A + B$$
$$1 = 10 + B$$
$$B = 1 - 10$$
$$B = -9$$

Substitute $B = -9$ and $A = 10$ in (ii)

$$0 = -4A - 3B + C$$
$$0 = -4(10) - 3(-9) + C$$
$$0 = -40 + 27 + C$$
$$0 = -13 + C$$
$$C = 13$$

$\therefore$ The resolve is $\frac{(n^2+1)}{(n-3)(n-2)^2} = \frac{10}{(n-3)} - \frac{9n+13}{(n^2-4n+4)}$

$$\Rightarrow \int \frac{n^2+1}{(n-3)(n-2)^2}\,dn = \int \frac{10}{n-3}\,dn - \int \frac{9n+13}{(n^2-4n+14)}\,dn$$

$$= 10\int \frac{1}{n-3}\,dn - \int \frac{9n}{n^2-4n+4}\,dn + \int \frac{13}{n^2-4n-}$$

$$= 10\ln(n-3 - 9\ln(n-2) + \frac{18}{(n-2)} - \frac{13}{(n-2}$$

$$= 10\ln(n-3) - 9\ln(n-2) + \frac{18-3}{(n-2)}$$

$$= 10\ln(n-3) - 9\ln(n-2) + \frac{18-13}{(n-2}$$

$$= 10\ln(n-3) - 9\ln(n-2) + \frac{5}{(n-2)} + C$$

$$\therefore \int \frac{n^2+1}{(n-3)(n-2)^2}\,dn = 10\ln(n-3) - 9\ln(n-2) + \frac{5}{(n-2)} + C$$

4 $\frac{(n^3 + n^2 + n + 1)}{n-1}\,dn$

Solution

$$\int \frac{(n^3 + n^2 + n + 1)}{(n-1)}\,dn = \int \frac{n^3}{n-1}\,dn + \int \frac{n^2}{(n-1)}\,dn + \int \frac{n}{n-1}\,dn + \int \frac{1}{n-1}\,dn$$

$= \frac{2n^3 + 3n^2 + 6n - 11}{6} + \ln(n-1) + \frac{n^2}{2} + n + \ln(n-1) + (n-1)$

$+ \ln(n-1) + \ln(n-1) + C$

$= \frac{2n^3 + 3n^2 + 6n - 11}{6} + \frac{n^2}{2} + n + n - 1 + \ln(n-1)$

$+ \ln(n-1) + \ln(n-1) + \ln(n-1) + C$

$= \frac{2n^3 + 3n^2 + 6n - 11 + 3n^2 + 6n + 6n - 6}{6} + 4\ln(n+1) + C$

$= \frac{2n^3 + 6n^2 + 18n - 17}{6} + 4\ln(n-1) + C$

$\therefore \int n^3 + \frac{n^2 + n + 1}{(n-1)}\, dn = \frac{2n^3 + 6n^2 + 18n - 17}{6} + 4\ln(n-1) + C$