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18/SCIO1/074

QUESTIONS

1). Explain with 2 examples what you understand by linear transformation.

2). Given a linear transformation of a matrix operator A on a vector x, compute T(x) if A=(1,9,3), (-2,6,7), (0, -1,3) and

x= 1

4

-8

3). Define completely with mathematical examples what you understand by Rank of a matrix.

<u>SOLUTION</u>

1. A linear transformation is a unction from one vector space to another that respects the underlying (linear) structure of each vector pace. It is also known as a linear operator or map.

<u>Examples</u>

(a.) The **expected value** of a random variable(which is in fact a function, and a member of a vector space)

E(X + Y) = E(x) + E(Y) and E[aX]= aE[X].

(b.) An identity map on any module is a linear transformation.

2. $\begin{pmatrix} 1 & 9 & 3 \\ -2 & 6 & 7 \\ 0 & -1 & 3 \end{pmatrix}$ X= $\begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$

T(x)= A(x)

T(x) = $1\begin{pmatrix} 1 \\ -2 \\ \end{pmatrix} +4 \begin{pmatrix} 9 \\ 6 \\ \end{pmatrix} -8 \begin{pmatrix} 3 \\ 7 \\ \end{pmatrix}$

0 -1 3

$$\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 36 \\ 24 \\ -4 \end{pmatrix} + \begin{pmatrix} -24 \\ -56 \\ -24 \end{pmatrix}$$

$= \begin{pmatrix} 13 \\ -34 \\ -28 \end{pmatrix}$ Hence, the transformation of $\begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$

3. The rank of a matrix is defined as the maximum number of linearly independent column vector in the matrix or maximum number of linearly independent roe vector in the matrix.

   Example

$$A = \begin{matrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{matrix}$$

(i.) <u>RANK OF A</u>

$$|A| = \begin{vmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix} -(-3) \begin{vmatrix} 4 & 2 \\ 8 & 1 \end{vmatrix} +6 \begin{vmatrix} 4 & 0N \\ 8 & 5 \end{vmatrix}$$

$= 1(0 - 10) + 3(4-16) + 6(20-0)$

$= -10 - 36 + 120$

$= 74$

Since $|A| \neq 0$, The Rank of A is 3

Note: But if the determinant of the matrix is equal to 0 then we delete arrow and column of the 3 x 3 matrix forming a 2 x 2 matrix then we find the determinant of the matrix. If we get an answer not equal to 0 then the matrix of rank 2