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DEPARTMENT: COMPUTER SCIENCE

MATRIC NO: 19/SCI01/015

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(1) $M = pi - 6j - 3K$

$N = 4i + 3j - K$

$O = i - 3j + 2K$

(a) M and N are perpendicular to each other

$$M \cdot N = (pi - 6j - 3K) \cdot (4i + 3j - K)$$
$$= 4p - 18 + 3$$
$$= 4p - 15$$

Since they are perpendicular to each other

$$4p - 15 = 0$$
$$\frac{\cancel{4}p}{\cancel{4}} = \frac{15}{4}$$
$$p = 15/4$$

(b) M, N and O are coplanar

$$M \cdot (N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$$= p(6 - (3)) + 6(8 - (-1)) - 3(-12 - 3) = 0$$
$$= 3p + 6(8 + 1) - 3(-15) = 0$$
$$= 3p + 54 + 45 = 0$$
$$= 3p + 99 = 0$$
$$\frac{\cancel{3}p}{\cancel{3}} = \frac{-99}{3}$$
$$p = -33$$

(2) $\bar{V} = (3i+2j+5k)+(2i-j+6k)+(5i+2j-3k)$

$\bar{V} = 10i+3j+8k$

$a_x = 10$, $a_y = 3$ and $a_z = 8$

$|V| = \sqrt{10^2+3^2+8^2}$

$|V| = \sqrt{100+9+64}$

$|V| = \sqrt{173} = 13.15$

(i) The direction cosines are

$\cos\alpha = \frac{a_x}{|V|} = \frac{10}{13.15} = 0.761$

$\cos\beta = \frac{a_y}{|V|} = \frac{3}{13.15} = 0.228$

$\cos\gamma = \frac{a_z}{|V|} = \frac{8}{13.15} = 0.608$

(ii) Unit vector

$e_v = \frac{V}{|V|} = \frac{10i+3j+8k}{13.15}$

(3) $F = 3ui + u^2 j + (u+2)k$

$V = 2ui - 3uj + (u-2)k$

$$(F\times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$= i\left[u^2(u-2)-[-3u(u+2)]\right] - j\left[3u(u-2)-2u(u+2)\right] + k\left[-9u^2-2u^3\right]$$

$$= i\left[u^3-2u^2-[-3u^2-6u]\right] - j\left[3u^2-6u-2u^2+4u\right] + k\left[-2u^3-9u^2\right]$$

$$= i\left[u^3-2u^2+3u^2+6u\right] - j\left[u^2-10u\right] + k\left[-2u^3-9u^2\right]$$

$$= i\left[u^3+u^2+6u\right] - j\left[u^2-10u\right] + k\left[-2u^3-9u^2\right]$$

$$\int_0^1 (F\times V) = \int i\left[u^3+u^2+6u\right] - \int j\left[u^2-10u\right] + \int k\left[-2u^3-9u^2\right]$$

$$= i\int u^3+u^2+6u - j\int u^2-10u + k\int -2u^3-9u^2$$

$$= i\left[\frac{u^4}{4}+\frac{u^3}{3}+\frac{\cancel{6}^{3}u^2}{\cancel{2}}\right] - j\left[\frac{u^3}{3}-\frac{\cancel{10}^{5}u^2}{\cancel{2}_1}\right] + k\left[\frac{-\cancel{2}^{1}u^4}{\cancel{4}_2}-\frac{\cancel{9}^{3}u^3}{\cancel{3}_1}\right]+c$$

$$= i\left[\frac{u^4}{4}+\frac{u^3}{3}+3u^2\right] - j\left[\frac{u^3}{3}-5u^2\right] + k\left[\frac{-u^4}{2}-3u^3\right]+c$$

$$\int_0^1 (F \times V) = i\left[\frac{(1)^4}{4}+\frac{(1)^3}{3}+3(1)^2\right] - j\left[\frac{(1)^3}{3}-5(1)^2\right] + k\left[\frac{-(1)^4}{2}-3(1)^3\right]+c$$

$$-\left[0+c\right]$$

$$\int_0^1 (F \times V) = i\left[\frac{1}{4}+\frac{1}{3}+3\right] - j\left[\frac{1}{3}-5\right] + k\left[\frac{-1}{2}-3\right]+\cancel{c}-\cancel{c}$$

$$\int_0^1 (F \times V) = i\left[\frac{43}{12}\right] - j\left[\frac{-14}{3}\right] + k\left[\frac{-7}{2}\right]$$

$$\int_0^1 (F \times V) = \frac{43}{12}i + \frac{14}{3}j - \frac{7}{2}k$$