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Computer Science

①  $\frac{3n-1}{(n-1)(n-2)(n-3)}$  pln

Solution

$$\frac{3n-1}{(n-1)(n-2)(n-3)} = \frac{A}{n-1} + \frac{B}{n-2} + \frac{C}{n-3}$$

Find the lcm of the righthand side

$$= \frac{A(n-2)(n-3) + B(n-1)(n-3) + C(n-2)(n-1)}{(n-1)(n-2)(n-3)}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$$3n-1 = A(n-2)(n-3) + B(n-1)(n-3) + C(n-1)(n-2)$$

let  $n=1$

$$3(1)-1 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$$

$$2 = A(-1)(-2) + B(0) + C(0)$$

$$\frac{2}{2} = \frac{2A}{2}$$

$$A=1$$

let  $n=2$

$$3(2)-1 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$$

$$5 = 0 + (-B) + 0$$

$$\frac{5}{-1} = \frac{-B}{-1}$$

$$B = -5$$

let  $n=3$

$$3(3)-1 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$$

$$0 + 0 + C(2)(1)$$

$$8 = 2C \quad C=4$$

$$\frac{8}{2} = \frac{2C}{2}$$

The residue is  $3(m-1)$

$$\frac{(m-1)(m-2)(m-3)}{m!} = \frac{1}{m!} - \frac{5}{m^2} + 4$$

$$\Rightarrow \int \frac{(Bm-1)}{(m-1)(m-2)(m-3)} dm = \int \frac{1}{(m-1)} dm - \int \frac{5}{m-2} dm + 4 \int \frac{dm}{m-3}$$

$$= \ln(m-1) - 5 \int \frac{1}{m-2} dm + 4 \int \frac{dm}{m-3}$$

$$= \ln(m-1) - 5 \ln(m-2) + 4 \ln(m-3) + C$$

(2)  $n^2 + n + 1$

$$\frac{(n^2 + n + 1)}{(n^2 + 1)(n^2 + 1)}$$

Solution

$$\int \frac{n^2 + n + 1}{(n^2 + 1)(n^2 + 1)} dn$$

Resolve

$$\frac{n^2 + n + 1}{(n^2 + 1)(n^2 + 1)} = \frac{A}{n^2 + 1} + \frac{Bn + C}{n^2 + 1}$$

Find the LCM of the right hand side

$$\frac{n^2 + n + 1}{(n^2 + 1)(n^2 + 1)} = \frac{A}{n^2 + 1} + \frac{Bn + C}{n^2 + 1}$$

$$\frac{(n^2 + 1)(n^2 + 1)}{(n^2 + 1)(n^2 + 1)}$$

Let  $n^2 + 1 = 2$

$$(-2)^2 + (-2) + 1 = A(-2)^2 + 1 + [B(-2) + C](2)$$

$$\frac{3}{5} = \frac{4A}{5} + \frac{B}{5}$$

$$A = 3/5$$

From the right hand side

$$A(n^2 + 1) + [Bn + C](n^2 + 1)$$

$$= An^2 + A + [Bn^2 + 2Bn + Cn + C]$$

$$= An^2 + A + Bn^2 + 2Bn + Cn + C$$

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$$= An^2 + Bn^2 + 2Bn + Cn + A + C$$

$$= n^2(A+B) + n(2B+C) + A+C$$

$$n^2 + n + 1 = n^2(A+B) + n(2B+C) + A+C$$

Compare the coefficients

$$1 = A + B \quad \dots (i)$$

$$1 = 2B + C \quad \dots (ii)$$

$$1 = A + 2C \quad \dots (iii)$$

Substitute  $A = 3/5$  in (i)

$$1 = \frac{3}{5} + B$$

$$B = 1 - \frac{3}{5}$$

$$B = \frac{2}{5}$$

Substitute  $B = 2/5$  in equation (ii)

$$1 = 2\left(\frac{2}{5}\right) + C$$

$$1 = \frac{4}{5} + C$$

$$C = 1 - \frac{4}{5}$$

$$C = \frac{1}{5}$$

$\therefore$  The residue is  $\frac{n^2 + n + 1}{(n+2)(n^2+1)} = \frac{3}{5(n+2)} + \frac{2n+1}{5(n^2+1)}$

$$\begin{aligned} \Rightarrow \int \frac{n^2 + n + 1}{(n+2)(n^2+1)} dn &= \int \frac{3}{5(n+2)} dn + \int \frac{2n+1}{5(n^2+1)} dn \\ &= \int \frac{3}{5(n+2)} dn + \frac{1}{5} \int \frac{2n+1}{n^2+1} dn \end{aligned}$$

$$= \frac{3}{5} \int \ln(n+2) + \frac{1}{5} \int \frac{2n+1}{n^2+1} dn$$

$$= \frac{3}{5} \ln(n+2) + \frac{1}{5} \int \ln(n^2+1) \operatorname{arctan} n$$

$$= \frac{3}{5} \ln(n+2) + \frac{1}{5} (n(n^2+1) +$$

$$\therefore \int \frac{n^2 + n + 1}{(n+2)(n^2+1)} dn = \frac{3}{5} \ln(n+2) + \frac{1}{5} \ln(n^2+1) + \frac{\operatorname{arctan}(n)}{5}$$

$$3 \int \frac{(n^2+1) \, dn}{(n-3)(n-2)^2}$$

solution

$$\int \frac{(n^2+1)}{(n-3)(n-2)^2} \, dn$$

Resolve

$$\frac{(n^2+1)}{(n-3)(n-2)^2} = \frac{(n^2+1)}{(n-3)(n^2-4n+4)}$$

$$\frac{(n^2+1)}{(n-3)(n^2-4n+4)}$$

$$= \frac{A}{(n-3)} + \frac{Bn+C}{(n^2-4n+4)}$$

Find the form of the R.H.S

$$\frac{(n^2+1)}{(n-3)(n^2-4n+4)} = \frac{A}{(n-3)} + \frac{Bn+C}{(n^2-4n+4)}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$$(n^2+1) = A(n^2-4n+4) + [Bn+C](n-3)$$

Let  $n=3$

$$(3)^2 + 1 = A((3)^2 - 4(3) + 4) + [B(3) + C](3-3)$$

$$10 = A(1) \quad \text{--- (i)}$$

$$A = 10$$

From the R.H.S

$$A(n^2-4n+4) + [Bn+C](n-3)$$

$$= An^2 - 4An + 4A + [Bn^2 - 3Bn + Cn - 3C]$$

$$= An^2 - 4An + 4A + Bn^2 - 3Bn + Cn - 3C$$

$$= n^2(A+B) + n(-4A-3B+C) + 4A-3C$$

Compare the coefficient

$$1 = A+B \quad \text{--- (i)}$$

$$0 = -4A - 3B + C \quad \text{--- (ii)}$$

$$1 = 4A - 3C \quad \text{--- (iii)}$$

Substitute  $A=10$  in equation (i)

$$1 = A + B$$

$$1 = 10 + B$$

$$B = 1 - 10$$

$$B = -9$$

Substitute  $B=-9$  and  $A=10$  in (ii)

$$0 = -4A - 3B + C$$

$$0 = -4(10) - 3(-9) + C$$

$$0 = -40 + 27 + C$$

$$0 = -13 + C$$

$$C = 13$$

∴ The residue is  $\frac{10}{(n-3)(n-2)^2} = \frac{10}{(n-3)(n-2)} - \frac{9n+13}{(n-2)^2}$

$$\begin{aligned} \Rightarrow \int \frac{n^2+1}{(n-3)(n-2)^2} dn &= \int \frac{10}{n-3} dn - \int \frac{9n+13}{(n^2-4n+4)} dn \\ &= 10 \int \frac{1}{n-3} dn - \int \frac{9n}{n^2-4n+4} dn + \int \frac{13}{n^2-4n} dn \\ &= 10 \ln(n-3) - 9 \ln(n-2) + \frac{18}{n-2} - \frac{13}{n-2} \\ &= 10 \ln(n-3) - 9 \ln(n-2) + \frac{18-13}{n-2} \\ &= 10 \ln(n-3) - 9 \ln(n-2) + \frac{5}{n-2} + C \end{aligned}$$

$$\therefore \int \frac{n^2+1}{(n-3)(n-2)^2} dn = 10 \ln(n-3) - 9 \ln(n-2) + \frac{5}{n-2} + C$$

$$(4) \int \frac{(n^3 + n^2 + n + 1) dn}{n-1}$$

Solution

$$\begin{aligned} \int \frac{(n^3 + n^2 + n + 1) dn}{(n-1)} &= \int \frac{n^3}{n-1} dn + \int \frac{n^2}{n-1} dn + \int \frac{n}{n-1} dn \\ &\quad + \int \frac{1}{n-1} dn \end{aligned}$$

$$\begin{aligned}
 &= 2n^3 + \frac{3n^2}{6} + 6n - 11 + \ln(n-1) + \frac{n^2 + n + \ln(n-1)}{2} + (n-1) \\
 &\quad + \ln(n-1) + \ln(n-1) + C \\
 &= 2n^3 + \frac{3n^2}{6} + 6n - 11 + \frac{n^2}{2} + n + n - 1 + \ln(n-1) \\
 &\quad + \ln(n-1) + \ln(n-1) + \ln(n-1) + C
 \end{aligned}$$

$$\begin{aligned}
 &= 2n^3 + \frac{3n^2}{6} + 6n - 11 + \frac{3n^2}{6} + 6n + 6n - 6 + 4 \ln(n-1) + C \\
 &= 2n^3 + \frac{6n^2}{6} + 18n - 17 + 4 \ln(n-1) + C
 \end{aligned}$$

$$\therefore \int \frac{n^3 + \frac{n^2}{6} + n + 1}{(n-1)} dn = 2n^3 + \frac{6n^2}{6} + 18n - 17 + 4 \ln(n-1) + C$$