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Assignment.

$$(1) \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$$

Solution.

$$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$$

Resolve

$$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

Find the LCM of the right hand side.

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

let $x=1$

$$3(1)-1 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$$

$$2 = A(-1)(-2) + B(0)(-2) + C(0)(-1)$$

$$2 = A(2) + 0 + 0$$

$$2 = A(-1)(-2) + B(0)(-2) + C(0)(-1)$$

$$\frac{2}{2} = \frac{2A}{2}$$

$$A = 1$$

$$3(2) - 1 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)$$

$$(2-2)$$

$$5 = A(0)(-1) + B(1)(-1) + C(-1)(0)$$

$$\frac{5}{-1} = \frac{-B}{-1}$$

$$B = -5$$

$$\text{let } x = 3$$

$$3(3) - 1 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)$$

$$(3-2)$$

$$8 = A(1)(0) + B(2)(0) + C(2)(1)$$

$$\frac{8}{2} = \frac{7C}{2}$$

$$C = 4$$

$$\therefore \text{The resolve is } \frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\begin{aligned} \Rightarrow \int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx &= \int \frac{1}{(x-1)} dx - \int \frac{5}{(x-2)} dx + \int \frac{4}{(x-3)} dx \\ &= \ln(x-1) - 5 \int \frac{1}{(x-2)} dx + 4 \int \frac{1}{(x-3)} dx \\ &= \ln(x-1) - 5 \ln(x-2) + 4 \ln(x-3) + C \end{aligned}$$

$$(2) \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$$

Solution

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$$

Resolve

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$$

Find the L.C.M of the right hand side

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1) + [(Bx+C)(x+2)]}{(x+2)(x^2+1)}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$$x^2+x+1 = A(x^2+1) + [(Bx+C)(x+2)]$$

Let $x = -2$

$$(-2)^2 + (-2) + 1 = A((-2)^2 + 1) + [(B(-2) + C)(-2 + 2)]$$

$$4 - 2 + 1 = 5A + 0$$

$$\frac{3}{5} = \frac{5A}{5}$$

$$A = \frac{3}{5}$$

From the R.H.S

$$A(x^2+1) + (Bx+C)(x+2)$$
$$= Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$$
$$= Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$$

Collect like terms

$$= Ax^2 + Bx^2 + 2Bx + Cx + A + 2C$$
$$= x^2(A+B) + x(2B+C) + A+2C$$

$$x^2 + x + 1 = x^2(A+B) + x(2B+C) + A+2C$$

Compare the coefficient

$$1 = A+B \text{ --- (i)}$$

$$1 = 2B+C \text{ --- (ii)}$$

$$1 = A+2C \text{ --- (iii)}$$

Substitute $A = \frac{3}{5}$ in equation (i)

$$1 = \frac{3}{5} + B$$

$$B = 1 - \frac{3}{5}$$

$$B = \frac{2}{5}$$

Substitute $B = \frac{2}{5}$ in equation (ii)

$$1 = 2\left(\frac{2}{5}\right) + C$$

$$1 = \frac{4}{5} + C$$

$$C = 1 - \frac{4}{5}$$

$$C = \frac{1}{5}$$

$$\therefore \text{The resolve is } \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$$

$$\begin{aligned}
\Rightarrow \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx &= \int \frac{3}{5(x+2)} dx + \int \frac{2x+1}{5(x^2+1)} dx \\
&= \int \frac{3}{5(x+2)} dx + \frac{1}{5} \int \frac{2x+1}{x^2+1} dx \\
&= \frac{3}{5} \int \frac{1}{x+2} dx + \frac{1}{5} \left[\int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx \right] \\
&= \frac{3}{5} \ln|x+2| + \frac{1}{5} \left[\int \frac{2x \cdot x^{-1} du}{u \cdot 2x} + \arctan(x) \right] \\
&= \frac{3}{5} \ln|x+2| + \frac{1}{5} \left[\int \frac{1}{u} du + \arctan(x) \right] \\
&= \frac{3}{5} \ln|x+2| + \frac{1}{5} \left[\ln|x^2+1| + \arctan(x) \right] \\
&= \frac{3}{5} \ln|x+2| + \frac{1}{5} \ln|x^2+1| + \arctan(x) + c
\end{aligned}$$

$$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx = \frac{3}{5} \ln|x+2| + \frac{1}{5} \ln|x^2+1| + \arctan(x) + c$$

3. $\int \frac{x^2+1}{(x-3)(x-2)^2} dx$

Solution

$$\int \frac{x^2+1}{(x-3)(x-2)^2} dx$$

Resolve

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{x^2+1}{(x-3)(x^2-4x+4)}$$

$$\frac{x^2+1}{(x-3)(x^2-4x+4)} = \frac{A}{x-3} + \frac{Bx+C}{x^2-4x+4}$$

Find the L.C.M of the right hand side

$$\frac{x^2+1}{(x-3)(x^2-4x+4)} = \frac{A(x^2-4x+4) + (Bx+C)(x-3)}{(x-3)(x^2-4x+4)}$$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$$(x^2+1) = A(x^2-4x+4) + [(Bx+C)(x-3)]$$

Let $x=3$

$$(3)^2+1 = A((3)^2-4(3)+4) + [(B(3)+C)(3-3)]$$

$$10 = A(1)$$

$$A = 10$$

From the R.H.S

$$A(x^2-4x+4) + [(Bx+C)(x-3)]$$
$$= Ax^2-4Ax+4A + [Bx^2-3Bx+Cx-3C]$$

$$= Ax^2-4Ax+4A+Bx^2-3Bx+Cx-3C$$

Collect like terms

$$= Ax^2+Bx^2-4Ax-3Bx+Cx+4A-3C$$

$$= x^2(A+B) + x(-4A-3B+C) + 4A-3C$$

$$(x^2+1) = x^2(A+B) + x(-4A-3B+C) + 4A-3C$$

compare the coefficient

$$1 = A+B \text{ ---- (i)}$$

$$0 = -4A-3B+C \text{ ---- (ii)}$$

$$1 = 4A-3C \text{ ---- (iii)}$$

Substitute $A=10$ in equation (i)

$$1 = A+B$$

$$1 = 10+B$$

$$B = 1-10$$

$$B = -9$$

Substitute $B=-9$ and $A=10$ in equation (ii)

$$0 = -4A-3B+C$$

$$0 = -4(10)-3(-9)+C$$

$$0 = -40+27+C$$

$$0 = -13+C$$

$$C = 13$$

$$\therefore \text{The residue is } \frac{(x^2+1)}{(x-3)(x-2)^2} = \frac{10}{(x-3)} - \frac{9x+13}{(x^2-4x+4)}$$

$$\begin{aligned}
\Rightarrow \int \frac{x^2+1}{(x-3)(x-2)^2} dx &= \int \frac{10}{x-3} dx - \int \frac{9x+13}{x^2-4x+4} dx \\
&= 10 \int \frac{1}{x-3} dx - \int \frac{9x}{x^2-4x+4} dx + \int \frac{13}{x^2-4x+4} dx \\
&= 10 \ln(x-3) - 9 \ln(x-2) + \frac{18}{x-2} - \frac{13}{x-2} \\
&= 10 \ln(x-3) - 9 \ln(x-2) + \frac{18-13}{x-2} \\
&= 10 \ln(x-3) - 9 \ln(x-2) + \frac{5}{x-2} + C
\end{aligned}$$

$$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2} dx = 10 \ln(x-3) - 9 \ln(x-2) + \frac{5}{x-2} + C$$

$$4) \int \frac{x^3+x^2+x+1}{x-1} dx$$

Solution

$$\int \frac{x^3+x^2+x+1}{x-1} dx = \int \frac{x^3}{x-1} dx + \int \frac{x^2}{x-1} dx + \int \frac{x}{x-1} dx + \int \frac{1}{x-1} dx$$

$$= \frac{2x^3+3x^2+6x-11}{6} + \ln(x-1) + \frac{x^2}{2} + x + \ln(x-1) + (x-1)$$

$$+ \ln(x-1) + \ln(x-1) + C$$

$$= \frac{2x^3+3x^2+6x-11}{6} + \frac{x^2}{2} + x + x-1 + \ln(x-1) + \ln(x-1) +$$

$$\ln(x-1) + \ln(x-1) + C$$

$$= \frac{2x^3+3x^2+6x-11+3x^2+6x+6x-6}{6} + 4 \ln(x-1) + C$$

$$= \frac{2x^3+6x^2+18x-17}{6} + 4 \ln(x-1) + C$$

$$\therefore \int \frac{x^3+x^2+x+1}{x-1} dx = \frac{2x^3+6x^2+18x-17}{6} + 4 \ln(x-1) + C$$