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18/Eng04/080
Electrical / Electronics Engineering
EEE

Linear Systems

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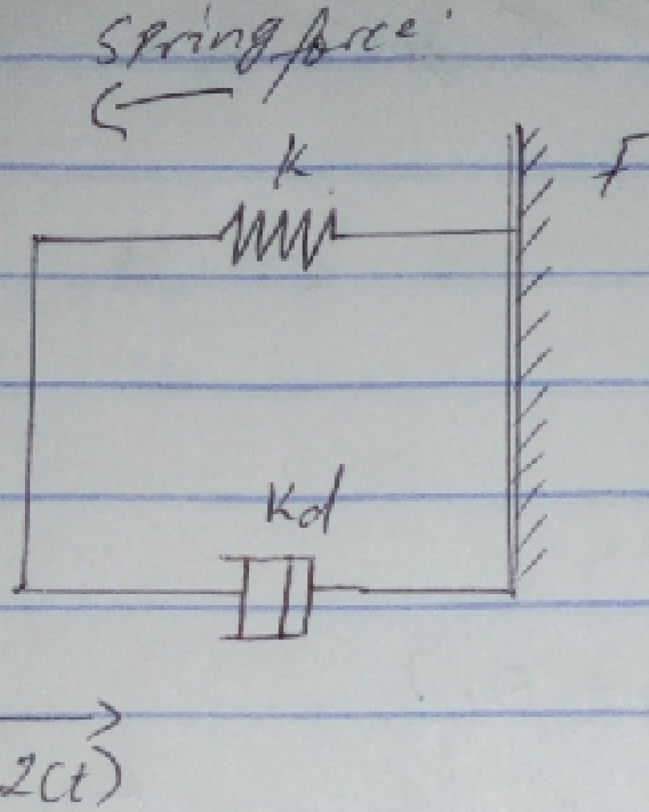
1.



Soln...

Spring $\rightarrow K(x-0)$

F(damper) $\rightarrow K_d \dfrac{d(x-0)}{dt}$

$F(t) \rightarrow F(t)$

from newton's law, : $F(t) - K(x-0) - K_d \dfrac{d(x-0)}{dt} = 0$

$\Rightarrow 0 = F(t) - Kx - K_d \dfrac{dx}{dt}$

laplace: $\mathcal{L}(F(s) - K x(s) - K_d s x(s))$

$\Rightarrow F(s) - K x(s) - K_d s\, x(s) = 0$

$F(s) = [K + K_d s]\, x(s)$

$G(s) = \dfrac{X(s)}{F(s)} = \dfrac{1}{K + K_d s}$

$$= \frac{1/K}{1 + [K_d/K] s}$$

from $\frac{1/K}{\tau s + 1}$

$$\Rightarrow \tau = \frac{K_d}{K} \equiv \frac{0.03}{4 \times 10^3}$$

$$= 0.75 \times 10^{-5} \, s$$

2

②

$$E_2 = mc \Delta\theta \equiv mc [\theta_2 - \theta_1]$$

$$E_1 = mc [\theta_2 - \theta_1]$$

θ = new temperature

$$G(s) = \frac{E_2}{E_1} = \frac{mc [\theta_2 - \theta_1]}{mc [\theta_2 - \theta_1]}$$

$$= \frac{1}{\tau s + 1}$$

$$\frac{\theta - \theta_1}{\theta_2 - \theta} = \frac{1}{\tau s + 1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{\tau s + 1}$$

Let $\theta_2 - \theta_1 = K(t)$

$$\Rightarrow (\theta_2 - \theta_1)(s) = \frac{K(t)}{\tau s + 1}$$

$$\Rightarrow [\theta_2 - \theta_1](s) = \frac{K(t) (1/\tau)}{s + 1/\tau}$$

laplace Δ of Kdt $= K/s$

$$\rightarrow [\theta - \theta_1]_{(s)} = \frac{K\,(1/T)}{s\,(s+1/T)}$$

$$\mathcal{L}^{-1}(\theta - \theta_1) = K(1 - e^{t/T})$$

③

$$\frac{w}{K_m x} = \frac{1}{Ts+1}$$

$T = 1/K_s$ $\qquad K_m = \frac{K_1 K_2}{K_s}$

$\therefore$ $w = \frac{K_m x}{Ts+1}$

Laplace of step input

$$w = \frac{K_m x}{s}\left(\frac{1}{Ts+1}\right)$$

$$\frac{K_m x}{s}\left(\frac{1/T}{s+1/T}\right)$$

$$w_{(s)} = K_m x\,(1 - e^{-t/T})$$

@ $t=0$; $K_m x\,(1 - e^{0}) = $ initial

@ $t=T$; $K_m x\,(1 - e^{-t/T}) = 0.63\,K_m x$

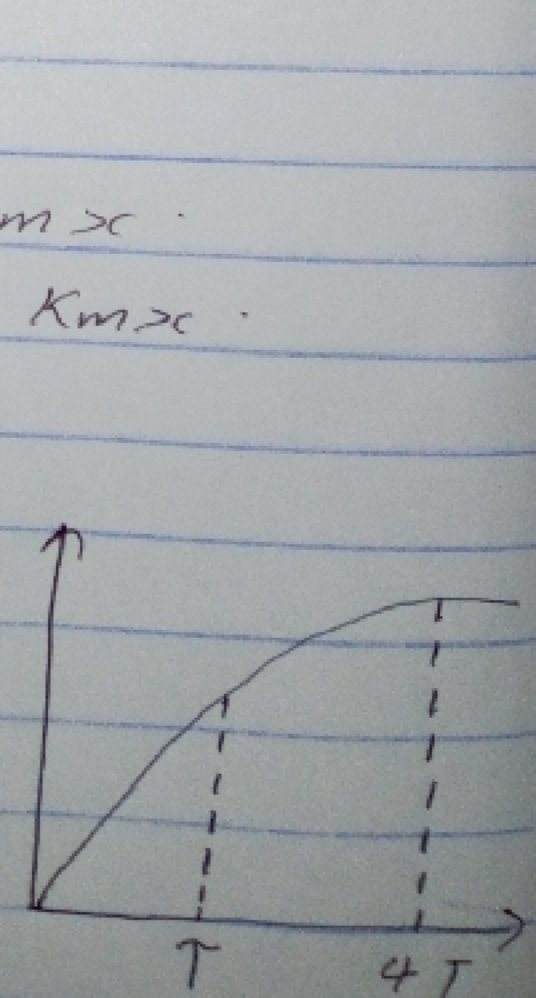
@ $t=4T$; $K_m x\,(1 - e^{-4t/T}) = 0.981\,K_m x$

for $t = T$

$\Delta\% = (0.632 - 0) \times 100\% = 63.2\%$

$t = 4t$

$\Delta\% = (0.981 - 0) \times 100\% = 98.1\%$



$\theta_1(t) = Ct$

$\theta_1(s) = \frac{C}{s^2}$

$\frac{\theta_o(s)}{\theta_1} \neq \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_1}{3s+1}$

$\therefore \rightarrow \theta_o(s) = \frac{C}{s^2} \div 3s+1$

$= \frac{C}{s^2(3s+1)}$

$\theta_o(t) = \frac{C/3}{3^2(3+1/3)}$

$\theta_o(t) = C[t-3(1-e^{-t/3})]$ —— (i)

@ t is lose

$\theta_o(t) = C[t-3(1)]$

$\theta_o(t) = Ct-3C$

$\theta_e = \theta_1 - \theta_o = Ct-[Ct-3C] = Ct-Ct-3C$

$= 3C$

$\tau = 3,\ C = 4\ mm/s$

after 2 sec

$\theta_1 = 4\times 2 = 8\ mm$

$\theta_e = C\times 3 = 4\times 3 = 12\ mm$

$\therefore \theta_o(t) = C[t-3(1-e^{-t/3})]$

$\rightarrow \theta_o = 4[2 - 3(1 - e^{-2/k})]$
$= 2.161\ mm$ //

5/

⑤

i) $\frac{2}{0.2s + 0.5} \equiv \frac{2/0.5}{\frac{0.2s}{0.5} + 1}$

$= \frac{4}{0.4s + 1}$ from $\frac{K}{\tau s + 1}$

$\therefore \Rightarrow 4 =$ D.C gain , $0.4 =$ time Constant.

ii) $\frac{0.2}{0.05s + 0.1} \equiv \frac{0.2/0.1}{\frac{0.05s}{0.1} + \frac{0.1}{0.1}}$

$= \frac{2}{0.5s + 1}$ from $\frac{K}{\tau s + 1}$

$\Rightarrow 2 =$ D.C gain , $0.5 =$ time Constant.

iii) $\frac{2}{3s + 1} \Rightarrow$ from $\frac{K}{\tau s + 1}$

$\Rightarrow 2 =$ D.C gain , $3 =$ time Constant.

iv) $\frac{16}{8s + 4} \equiv \frac{16/4}{8/4 s + 4/4}$

$= \frac{4}{2s + 1}$ from $\frac{K}{\tau s + 1}$

$\therefore \Rightarrow 4 =$ D.C gain , $2 =$ time Constant.

6

(6)

$$\frac{\omega}{\theta}(s) = \frac{K_m}{T_m s + 2}$$

$K_m = 1.5^{st1}$

$T_m = 4$

$$\Rightarrow \frac{15}{4s+2} = \frac{15/2}{4s/2 + 1}$$

$$= \frac{7.5}{2s+1}$$

$\therefore \Rightarrow$, ~~[illegible]~~

6

(6)

$$\frac{\omega}{\theta}(s) = \frac{K_m}{T_m s + 2}$$

$K_m = 1.5^{st1}$, $T_m = 4$

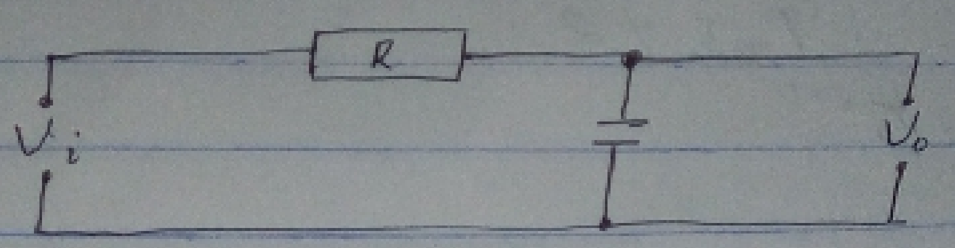
$$\Rightarrow \frac{15}{4s+2} = \frac{15/2}{4s/2 + 1}$$

$$= \frac{7.5}{2s+1}$$

$\therefore \Rightarrow$, 7.5^{st1} = D.C gain , 2 = time Constant.

System response 2.

(1).



$T = RC$

$R = 47\Omega, \quad C = 20\mu F, \quad V_i = 5\sin(2000t)$

$T = 47 \times 20 \times 10^{-6} \simeq 9.4 \times 10^{-9}$.

$$\left(\frac{V_o}{V_i}\right)(s) = \frac{1}{Ts+1} \quad ; \quad G(s) = \frac{1}{Ts+1}$$

$$G(\omega) = \frac{1}{9.4\times10^{-9}\, j\omega + 1} \times \frac{9.4\times10^{-9}\, j\omega - 1}{9.4\times10^{-9}\, j\omega - 1}$$

$$G(\omega) = \frac{9.4\times10^{-9}\, j\omega - 1}{(9.4\times10^{-9})^2\omega^2 - 1}$$

$$= \frac{-1}{(9.4\times10^{-9})^2\omega^2 - 1}$$

$\omega = 2000$ rad/s.

$$\phi = \tan^{-1}\left(\frac{9.4\times10^{-1}(2000)^2 - 1}{1}\right)$$

$$\phi = -61.99$$

$$|G(t\omega)| = \frac{1}{\sqrt{(9.4\times10^{-9})^2\omega^2 + 1^2}}$$

$$= \frac{1}{\sqrt{(9.4\times10^{-9})^2(2000)^2 + 1^2}} = 0.4696$$

$V_o = 5 \times 0.4696$

$\simeq 2.35$

(2)

$$\frac{x_0}{x_1} = \frac{1}{T^2 s^2 + 2\partial T s + 1}$$

$$G(s) = \frac{1}{(1 - T^2 s^2) + 2\partial T s}$$

$$G(jw) = \frac{1}{(1 - T^2 w^2) + 2\partial T j w}$$

$\partial = 0.2$, $T = 0.45$, $w = 2\partial$ rad/s

$$\Rightarrow G(jw) = \frac{1 - T^2 w^2 - 2\partial T j w}{(1 - T^2 w^2) + 4\delta^2 T^2 w^2}$$

$$= \frac{1 - (0.4)^2 (2.5)^2 - 2(0.2)(0.4)(2.5)}{(1 - (0.4)^2 (2.5)^2) - 4(0.2)^2 (0.4)(2.5)^2}$$

$G(jw) \equiv 0 = 2.5$

$$\phi = \tan^{-1}\left(\frac{2.5}{0}\right) = 0$$

$\tan^{-1}(\delta = 90°$

$$|Gjw| = \sqrt{0^2 + (2.5^2}$$

$= 2.5$

amplitude $= 6 \times 2.5$

$= 15$ //