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1.) $\frac{(3x-1)}{(x-1)(x-2)(x-3)}dx$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

Multiply both sides by $(x-1)(x-2)(x-3)$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

$3x-1 = Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3C + 2C$

$3x-1 = (A+B+C)x^2 + (-5A-4B-3C)x + (6A+3B+2C)$

$-1 = 6A+3B+2C$ — (i)

$3 = -5A-4B-3C$ — (ii)

$0 = A+B+C$ —— (iii)

$A = -B-C$ —(iv)

Substitute (iv) into (i) and (ii)

$6(-B-C) + 3B + 2C = -1$

$-5(-B-C) - 4B - 3C = 3$

$-3B - 4C = -1$ — (v)

$B + 2C = 3$ — (vi)

From (vi)

$B = 3 - 2C$

Substitute B into (v)

$-3(3-2C) - 4C = -1$

$-9 + 6C - 4C = -1$

$\frac{2C}{2} = \frac{8}{2}$

$C = 4$

Substitute C into (v)

$-3B - 4(4) = -1$

$-3B - 16 = -1$

$\frac{-3B}{-3} = \frac{15}{-3}$

$B = -5$

Substitute B and C into (i)

$6A + 3(-5) + 2(4) = -1$

$6A - 15 + 8 = -1$

$\frac{6A}{6} = \frac{6}{6}$

$A = 1$

$\therefore (A, B, C) = (1, -5, 4)$

$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} + \frac{(-5)}{(x-2)} + \frac{4}{(x-3)}$

$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \frac{1}{x-1} dx - \int \frac{5}{x-2} dx + \int \frac{4}{x-3}$

$= \ln(|x-1|) - 5\ln(|x-2|) + 4\ln(|x-3|) + C$

② $\frac{(x^2+x+1)}{(x+2)(x^2+1)} dx$

$\frac{(x^2+x+1)}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$

Multiply both sides by $(x+2)(x^2+1)$

$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

~~$x^2+x+1 = Ax^2+1 + Bx+2$~~

$x^2+x+1 = (A+B)x^2 + (2B+C)x + A+2C$

$1 = A+2C$ — (i)

$1 = C+2B$ — (ii)

$1 = A+B$ — (iii)

From $4(i)$

$A = 1 - B \quad - 4(v)$

Substitute $(v)$ into $1 \alpha (ii)$

$\cancel{1 - B + 2C = 1}$

$\div B + 2C \;=\; 1 - V$

3.) $\dfrac{x^2+1}{(x-3)(x-2)^2}$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

Multiply both sides by $(x-3)(x-2)^2$

$$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$$

$$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$$

$$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$$

$$x^2+1 = (A+B)x^2 + (-4A - 5B + C)x + (4A + 6B - 3C)$$

$$1 = 4A + 6B - 3C \quad -(i)$$

$$0 = -4A - 5B + C \quad -(ii)$$

$$1 = A + B \quad -(iii)$$

From $(iii)$

$A = 1 - B \quad -(iv)$

Substitute (IV) into (I) & (II)

$$4(1-B)+6B-3C=1$$
$$-4(1-B)-5B+C=0$$

$$4-4B+6B-3C=1$$
$$-4+4B-5B+C=0$$

$$2B-3C=-3 \quad \text{—— (V)}$$
$$-B+C=4 \quad \text{—— (VI)}$$

From (VI)

$$C=4+B \quad \text{—(VII)}$$

Substitute C into V

$$2B-3(4+B)=-3$$
$$2B-12-3B=-3$$
$$-B=9$$
$$B=-9$$

Substitute B into VII

$$C=4+B$$
$$C=4-9$$
$$C=-5$$

Substitute B and C into (I)

$$4A+6(-9)-3(-5)=1$$
$$4A-54+15=1$$
$$4A=1+39$$
$$4A=40$$
$$\frac{4A}{4}=\frac{40}{4}$$
$$A=10$$

$$\therefore (A,B,C)=(10,-9,-5)$$

$$\therefore \frac{x^3+1}{(x-3)(x-2)^2}=\frac{10}{(x-3)}+\frac{(-9)}{x-2}+\frac{(-5)}{(x-2)^2}$$

$$\therefore \int\frac{10}{x-3}\,dx-\int\frac{9}{x-2}\,dx-\int\frac{5}{(x-2)^2}\,dx$$

$$=10\ln(|x-3|)-9\ln(|x-2|)+\frac{5}{x-2}+C$$

4.) $\int \frac{x^3 + x^2 + x + 1}{x - 1} \, dx$

$\int \frac{x^3}{x-1} + \frac{x^2}{x-1} + \frac{x}{x-1} + \frac{1}{x-1} \, dx$

$\int \frac{x^3}{x-1} \, dx \int \frac{x^2}{x-1} \, dx \int \frac{x}{x-1} \, dx \int \frac{1}{x-1} \, dx$

$\frac{2x^3 + 5x^2 + 6x - 11}{} + \ln(|x-1|) + \frac{x^2}{2} + x + (|x-1|) + x - 1 + \ln(|x-1|) +$

$\ln(|x-1|) + C$

$= \frac{2x^3 + 3x^2 + 6x - 11}{6} - 4(\ln(|x-1|)) + C$