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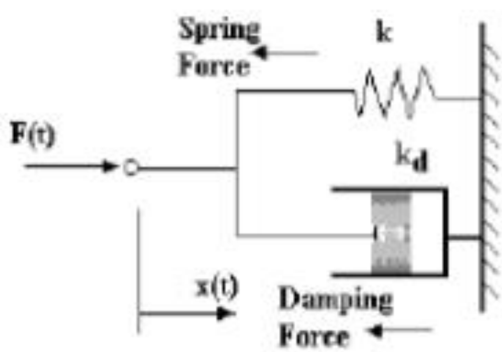
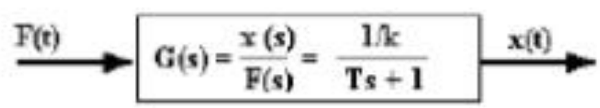
17/ENG04/057
ELECTRICAL ELECTRONICS ENGINEERING
EEE324 LINEAR SYSTEMS

QUESTION:

Homework

1. Show the derivation of the transfer function for spring and damper system shown. Given that the damping coefficient k_d is 0.03 and the spring stiffness k is 4 kN/m, determine the time constant for the system. (Answer 7.5 μs)
If a force of F = 100 N is suddenly applied, calculate the value of x after T seconds. (Answer 16 mm)





2. A block of metal has a mass of 0.5 kg, specific heat capacity 346 J/kg K and temperature of $\theta_1 = 20$°C. It is dropped into a large tank of oil at $\theta_2 = 120$°C and it is found that the temperature of the block takes 6 minutes to reach 119 °C.
Assume that the temperature of the block is changes by the law $\frac{\theta_1}{\theta_2}(s) = \frac{1}{(Ts+1)}$
Show that the temperature of the block changes with time by the law $\theta = \theta_1 + (\theta_2 - \theta_1)(1-e^{-t/T})$
Determine the time constant T and hence the thermal resistance between the block and the oil.
(Answer R=0.452 K/W)

3. A hydraulic motor has a nominal displacement of k_1 m^3/radian. The speed ω is controlled by a simple valve such that the pressure to the motor is k_2x where x is the input position of the valve.
The motor has a moment of inertia J kg m^2 and a damping coefficient of k_3 Nm s/radian.
Given that the torque developed by the motor is k_1p, show that the open loop transfer function

relating output ω to input x is given by $\frac{\omega}{k_m x} = \frac{1}{Ts + 1}$

$$T = \frac{J}{k_3} \quad \text{and} \quad k_m = \frac{k_1 k_2}{k_3}$$

The input is given a step change. Sketch the response of the output. Determine the % change in the output at $t = T$ and $t = 4T$. (63.2% and 99.9%)

Show on the sketch the affect of increasing the moment of inertia.

4. A position control system has a transfer function $G(s) = 1/(3s + 1)$. The input is changed at a constant rate of 4 mm/s from the zero position. Calculate the error after 2 seconds and the steady state error. (2.161 mm and 12 mm)

5 Find the D.C. gain and time constant for the following transfer functions.

i. $G(s) = \frac{2}{0.2s + 0.5}$ (4 and 0.4)

ii. $G(s) = \frac{0.2}{0.05s + 0.1}$ (2 and 0.5)

iii. $G(s) = \frac{2}{3s + 1}$ (2 and 3)

iv. $G(s) = \frac{16}{8s + 4}$ (4 and 2)

6 The output speed of a motor (ω rad/s) is related to the angle of the input sensor (θ radian) by the transfer function $\frac{\omega}{\theta}(s) = \frac{k_m}{T_m s + 2}$

Where $k_m = 15\ s^{-1}$ and $T_m = 4$ s

Determine the D.C. gain and time constant of the system. (7.5 s^{-1} and 2 seconds)

HOMEWORK

1. An electrical circuit has a resistor and capacitor has shown. Show that the transfer function is;

$(V_o/V_i)(s) = 1/(Ts+1)$ where $T = RC$

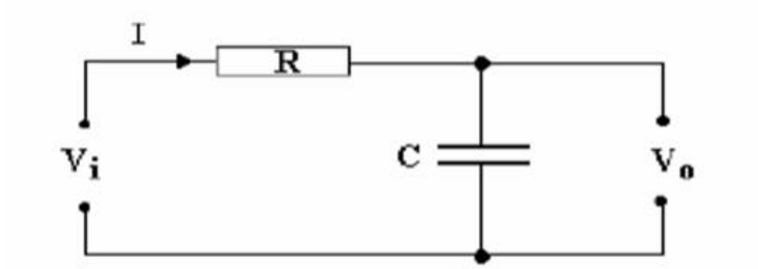


Figure 9

Given that $R = 47\ \Omega$ and $C = 20\mu F$ determine the output voltage when the input is sinusoidal such that $v_i = 5 \sin(2000t)$.

Answer 2.35sin(2000t - 62°)

2. A standard second order system has the transfer function $x_o/x_i = 1/(T^2s^2 + 2\delta Ts + 1)$

The time constant T is 0.4 Seconds and the damping ratio $\delta = 0.2$. The input is varied harmonically as $\theta_i = 6 \sin(\omega t)$ at 2.5 rad/s. Calculate the phase shift and amplitude of the output.

(Answer 90° and 15)

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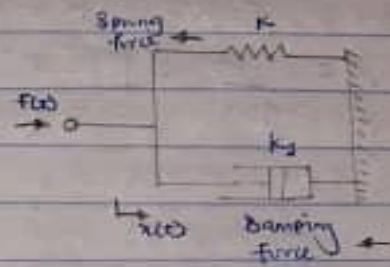
ASSIGNMENT SOLUTION

SYSTEM RESPONSE 1

1

Q1.

Show the Derivation of the transfer function;



Recall;

Spring force $f = Kx$

Damping Force $F = K_d \frac{dx}{dt}$

$\therefore F(t) = Kx + K_d \frac{dx}{dt}$

$F(s) = KX + K_d sX$

$\therefore F(s) = X(K + K_d s)$

$\therefore \frac{X}{F(s)} = \frac{1}{K + K_d S}$

$\therefore \frac{X}{F}(s) = \frac{1/K}{1 + (K_d/K)s} = \frac{1/K}{(K_d/K)s + 1}$

Given that;

$\frac{X}{F}(s) = \frac{1/K}{Ts+1} = \frac{1/K}{(K_d/K)s + 1}$ $\therefore T = K_d/K$.

Given that; $K_d = 0.03$, $K = 4KN/m$.

$\therefore T = 0.03/4000$

$T = 7.5 \times 10^{-6}$ seconds $\approx 7.5\mu s$

Calculate the Value of x after T sec

$\frac{X(s)}{F(s)} = \frac{1/K}{Ts+1}$

$$X(s) = \frac{F/K\tau}{s + 1/\tau}$$

From the table;

$F = F(s)$; F/s step Function

$$x = \frac{F/K\,(1/\tau)}{s(s + 1/\tau)}$$

$\therefore \quad a = 1/\tau$

$\therefore \quad X = F/K\,[1 - e^{-t/\tau}]$

$F = 100, \; K = 4000$

$\therefore \quad X = \frac{100}{4000} * [1 - e^{-1}]$

$\Rightarrow 0.0158$

$\Rightarrow 0.016$

$= 16\,mm$

Q 2

Given ; $\theta_1 = 20°c$, $\theta_2 = 120°c$ $m = 0.5kg$ $Hc = 3465 J/kg\,K$ $\Delta\theta = 1192$

Where $\theta_2(s) = 1/s$ step function

$\therefore$ From equation given;

$$\frac{\theta_1}{\theta_2}(s) = \frac{1}{\tau s + 1}$$

$$\theta(s) = \frac{\theta_2}{\tau s + 1}$$

$$\theta(s) = \frac{\theta_2}{s(\tau s + 1)}$$

$$\therefore \quad \theta(t) = \frac{1}{s(\tau s + 1)}$$

$$\therefore \quad \theta(t) = \frac{1/\tau}{s(s + 1/\tau)}$$

From the table ; $\theta(t) = 1 - e^{-t/\tau}$

Where; $\Delta\theta = \theta_2 - \theta_1$

2

$\therefore$ step change in $\Delta\theta$

$\Delta\theta \Rightarrow (\theta_2 - \theta_1)(1 - e^{-t/\tau})$

where; Initial temperature $= \theta_1$

$\therefore$ Representing change in θ

$\Rightarrow \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/\tau})$

Given that

$\theta(t) = \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/\tau})$

$\therefore$ $\theta(t) = 119 \quad \theta_1 = 20 \quad , \theta_2 = 120 \ , t = 6.$

$\therefore \quad 119 = 20 + (120 - 20)(1 - e^{-6/\tau}) = 20 + 10(1 - e^{-6/\tau})$

$119 = 20 + (100)(1 - e^{-6/\tau})$

$119 - 20 = 100(1 - e^{-6/\tau})$

$99 = 100(1 - e^{-6/\tau})$

$0.99 = 1 - e^{-6/\tau}$

$1 - 0.99 = e^{-6/\tau}$

$0.01 = e^{-6/\tau}$

$\ln 0.01 = -6/\tau$

$-4.605 = -6/\tau$

$\tau = -6/-4.605$

$\tau = 1.302$ minute

Recall ; Thermal Capacitance $= C = m * c$

$m = 0.5 \quad , c = 346$

$\therefore \quad C = 0.5 * 346$

$= 173 J/K$

Using the formula for thermal resistance

$\therefore \quad \tau = RC$ make R subject.

$R = \tau/C$

$R = \dfrac{1.302 \times 60}{173} = 0.452 K/W$

3

(3) Given that the torque developed = $K_1 P$

∴ At no load;

$T = K_1 P = J\dot{\theta} + K_3 \omega$

∴

$K_1 K_2 x(s) = Js\omega + K_3\omega$

Divide throug by K_3.

$$\left(\frac{K_1 K_2}{K_3}\right) x_{(s)} = \left(\omega\left(\frac{J}{K_3}\right)s + 1\right)$$

∴ $\cancel{K_1 K_2 \omega + \frac{d\omega}{dt}}$

$K_1 K_2 x = J d\omega/dt + K_3\omega$

∴ $K_1 K_2 x(s) = \omega(Js + K_3$

$K_m x(s) = \omega\{Ts + 1)\}$

$$\therefore \frac{\cancel{\omega}}{K_m x} = \frac{1}{Ts + 1}$$

$$\therefore \frac{\omega}{K_m} = \frac{x}{Ts + 1}$$

where $x(s) \rightarrow$ ~~Impulse~~ unit step

$x(s) = 1/s$

$$\therefore \frac{\omega}{K_m} = \frac{1}{s(Ts + 1)}$$

$$\therefore \frac{\omega}{K_m}(t) = \frac{1/T}{s(s + 1/T)}$$

$$\frac{\omega}{K_m} = 1 - e^{-t/t}$$

$$\therefore \omega = K_m(1 - e^{-t/T})$$

∴ at $t = T$ $t = 4T$

Put in eqration above;

$\therefore \quad w = Km(1 - e^{-t/\tau})$

$t = \tau$

$w = Km(1 - e^{-1})$

$w = 0.632\,Km$

$w = \underline{63.2\%}$

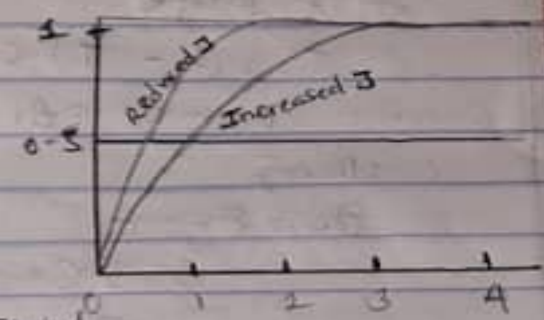
$\therefore \quad W = Km(1 - e^{-t/\tau}) \qquad t = 4\tau$

$W = Km(1 - e^{-4\tau/4\tau})$

$W = Km(1 - e^{-4})$

$W = 0.982\,Km$

$W = \underline{98.2\%}$



COMMENT:

It is observed from the plotted graph that the lower (Reduced value of τ takes shorter period to respond.

(4). Recall;

$$G(s) = \frac{1}{\tau s + 1}$$

$$\therefore \quad \frac{\theta_o}{\theta_i} = \frac{1}{\tau s + 1}$$

$$\theta_o = \frac{\theta_i}{\tau s + 1}$$

$\therefore$ Where; $\theta_i = c/s^2$

$$\therefore \quad \theta_o = \frac{c}{s^2(\tau s + 1)}$$

$$\theta_o = \frac{c(1/\tau)}{s^2(s + 1/\tau)}$$

Where; $c = K \; ; \; 1/\tau = a$

$$\theta_o = \frac{Ka}{s^2(s+a)}$$

$$\theta_o = c\{t - \tau(1 - e^{-t/\tau})\}$$

4

at time $t = 2$ seconds.

$\therefore$ $c = 4$ mm/s, $T = 3$ sec.

$\theta_o = c\{t - T(1 - e^{-t/T})\}$.

$\theta_o = 4\{2 - 3(1 - e^{-2/3})\}$.

$\theta_i = 2.16$mm.

$\therefore$ Recall from class;

at ~~t~~ large t, $e^{-t/T}$ is negligible.

$\therefore$ $\theta_o = (t - T)c$

$\therefore$ $ct - cT = \theta_o$

$\therefore$ where;

$\theta_e = \theta_o - \theta_i$

$\therefore$ $\theta_e = Ct - CT - Ct$

$\theta_e = CT$

$\therefore$ Error $\Rightarrow 4 * 3$

$= 12$mm.

③ Obtaining DC gain;

(i) $G(s) = \dfrac{2}{0.2s + 0.5}$ 5

Multiply thro by 2.

$G(s) = \dfrac{2 \times 2}{0.2s \times 2 + 0.5 \times 2}$

$G(s) = \dfrac{4}{0.4s + 1}$

$G(s) = \dfrac{4}{0.4s + 1}$

$\therefore$ DC gain $= 4$ time constant $= 0.4$

ii) $G(s) = \dfrac{0.2}{0.05s + 0.1}$

Multiply thru by 10

$G(s) = \dfrac{2}{0.5s + 1}$

$\therefore$ DC gain = 2 ; time Constant = 0.5

iii) $G(s) = \dfrac{2}{3s+1}$

DC gain = 2 ; time Constant = 3.

iv) $G(s) = \dfrac{16}{8s+4}$

Divide thru by 4

$\Rightarrow \dfrac{4}{2s+1}$

$\therefore$ DC gain = 4 ; time Constant = 2

6) Given the transfer function;

$w/\theta\,(s) = \dfrac{K_m}{T_m s + 2}$

$K_m = 15$ $\quad T_m = 4s$ Insert Values.

$\therefore \dfrac{w}{\theta} = \dfrac{15}{4s+2}$

Divide thru by 2.

$\dfrac{\omega}{\theta} = \dfrac{7.5}{2s+1}$

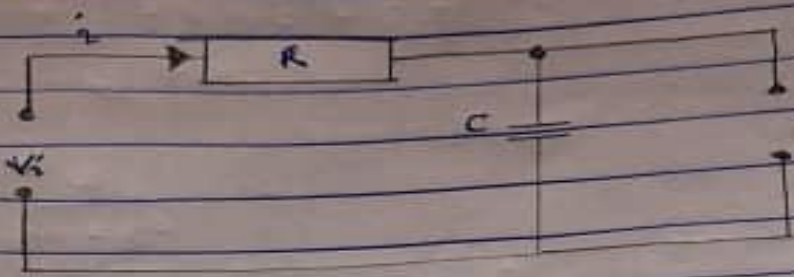
$\therefore$ DC Gain = 7.5 , time Constant = 2

6

SYSTEM RESPONSE 2

Q1

3



2 1

Preamble;

$R = 47\Omega$ $C = 20\mu F$ $V_o = ??$ $V_i = 5\sin(2000t)$.

$\therefore \omega = 2000$

Given the transfer function;

$$\frac{V_o}{V_i}(s) = \frac{1}{Ts+1}$$ where; $T = R * C$

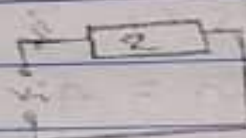
Resolving the circuit;

$R(t) = R(s)$

$C(t) = 1/C\,(s)$

$\therefore$ Total impedance $Z = R + 1/C(s)$

$= R + 1/C\cdot(s)$



$\therefore$ Voltage across this impedance

$V_i = I R$

$\therefore V_i = I Z$

$V_i = I(R + 1/C(s))$

Voltage Across the Capacitance ; Vo

$$V_o = I * \left(\frac{1}{C}\right)(s).$$

$$\therefore \frac{V_o}{V_i} = \frac{I(1/C\,(s))}{I(R + 1/C)}$$

$\therefore \frac{V_o}{V_i} = \frac{1}{RC(s)+1}$

Recall's in transfer function; RC = time Constant T

$\therefore \frac{V_o}{V_i} = \frac{1}{Ts+1}$

$\therefore T = RC$

$T = 47 * 20 \times 10^{-6}$

$T = 940 \mu s$ Time Constant

$\therefore \phi = -\tan^{-1}(\omega T)$

$= -\tan^{-1}(2000 \times 940 \times 10^{-6})$

$\Rightarrow -62^\circ$ phase Angle.

$\therefore \frac{\theta_o}{\theta_i} = \frac{1}{\sqrt{1+T^2\omega^2}}$

$= \frac{1}{\sqrt{1+(940 \times 10^{-6})^2 * 2000^2}}$

$= 0.47$

$\therefore$ Ampli $= 5 * 0.47$

Amplitude $= 2.35$

$\therefore \underline{\theta_o(t) = 2.35 \sin(200t - 62^\circ)}$

Q2

② Standard 2nd order transfer function. X_o/X_i

$\frac{X_o}{X_i} = \frac{1}{T^2s^2 + 2\delta Ts + 1}$

$T = 0.4$ $\delta = 0.2$ $\theta_i = 6\sin(\omega t)$ $\omega = 2.5$ rad/s.

$\therefore A = (1 - T^2\omega^2)$

$= 1 - (0.4^2 * 2.5^2)$

$A = 0$

$B = 2 * T * S * \omega$

$= 2 * 0.4 * 0.2 * 2.5$

$= 0.4$

$\therefore C = \dfrac{A}{A^2 + B^2}$

$= \dfrac{0}{0 + 0.4^2} = 0$

$D = \dfrac{B}{A^2 + B^2} = \dfrac{0.4}{0^2 + 0.4^2} = 2.5$

$\therefore$ phase Shift.

$\phi = -\tan^{-1} D/C$

$= -\tan^{-1} (2.5/0)$

$= -\tan^{-1} (\text{Inf})$

$= -90^\circ$

$\theta = \underline{\underline{90^\circ}}$

$\therefore$ Amplitude of the Output.

$\dfrac{\theta_o}{\theta_i} = \sqrt{C^2 + D^2}$

$\dfrac{\theta_o}{\theta_i} = \sqrt{0^2 + 2.5^2}$

$= 2.5$

$\therefore$ Amplitude;

$\theta_o = 2.5 * 6$

$= \underline{\underline{15}}$

$\therefore$ Phase Shift $= 90^\circ$ Amplitude $= 15$