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Computer Science

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If $M = Pi - 6j - 3k$, $N = 4i + 3j + k$; $O = i + 3j + 2k$, find the Value of P for which (a) M and N are perpendicular to each other (b) M, N and O are Coplanar

$M = Pi - 6j - 3k$, $N = 4i + 3j + k$, $O = i + 3j + 2k$

$M \cdot N = 0$

$(Pi - 6j - 3k) \cdot (4i + 3j + k) = 0$

$4P - 18 + 3 = 0$

$4P = 18 - 3 = 5$

$\frac{\cancel{4}P}{\cancel{4}} = \frac{15}{4} = 3.75$

(b) $M \cdot (N \times O) = 0$

$(N \times O) = (4i + 3j + k) \times (i + 3j + 2k)$

$$\bar{N} \times \bar{O} = \begin{vmatrix} i & j & k \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = i(6-3) - j(8-(-1)) + k(-12-3)$$

$= 3i - 9j - 15k$

$M \cdot (N \times O) = (Pi + 6j - 3k) \cdot (3i - 9j - 15k)$

$= 3p + 54 + 45 = 0$

$= 3p = -54 - 45 = -99$

$\frac{\cancel{3}p}{\cancel{3}} = \frac{-99}{3}$

$P = -33$

find the direction cosines and the unit Vector along the Sum of $3i + 2j + 5k$, ~~$2j +$~~ $2i - j + 6k$ and $5i + 2j - 3k$

Solution

let, $3i + 2j + 5k = a$

$2i - j + 6k = b$

$5i + 2j - 3k = c$

$a + b + c = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

= ~~collect like terms~~

$= \cancel{3i + 2i + 5i}\ 10i + 3j + 8k$

$|a + b + c| = \sqrt{(10)^2 + (3)^2 + (8)^2}$

$= \sqrt{173}$

Therefore, unit Vector $= U = \dfrac{a + b + c}{|a + b + c|}$

$= \dfrac{10}{\sqrt{173}} i + \dfrac{3}{\sqrt{173}} j + \dfrac{8}{\sqrt{173}} k$

Direction Cosines $= \cos\alpha = \dfrac{10}{\sqrt{173}}$ $\quad \cos\beta = \dfrac{3}{\sqrt{173}}$ $\quad \cos\gamma = \dfrac{8}{\sqrt{173}}$

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3

$f = 3ui + u^2 j + (u+2)k$

$v = 2ui - 3uj + (u-2)k$, evaluate the integral of $(f \times v)du$ from 0 to 1

$$(f \times v) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i((u^3 - 2u^2) + 3u^2 + 6) - j((3u^2 - 6u) - 2u^3) + k(-9u^2 - 2u^3)$$

$$= (u^3 + u^2 + 6)i - (-2u^3 + (3u^2 - 6u))j + (-9u^2 - 2u^3)k$$

$$\int_0^1 (f \times v)\,du = \int (u^3 + u^2 + 6)i + \int (2u^3 - 3u^2 + 6u)j + \int (-9u^2 - 2u^3)k$$

$$= \left[\frac{u^4}{4} + \frac{u^3}{3} + 6u\right]_0^1 + \left[\frac{u^4}{2} - u^3 + 3u^2\right]_0^1 + \left[-3u^3 - \frac{u^4}{2}\right]_0^1$$

$$= \left(\frac{1}{4} + \frac{1}{3} + 6\right) + \left(\frac{1}{2} - 1 + 3\right) + \left(-3 - \frac{1}{2}\right)$$

$$= \frac{79}{12} + \frac{5}{2} - \frac{7}{2} = \frac{29}{12} - \frac{1}{2} = \frac{79}{12} - \frac{12}{12}$$

$$\int_0^1 (f \times v)\,du = \frac{67}{12} = 5.5833$$

$$\int_0^1 (f \times v)\,du = \frac{72}{12} + \frac{5}{2} - \frac{7}{2}$$

$$\frac{72 + 30 - 42}{12} = \frac{60}{12} = \cancel{5.5833}\ 5$$