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### Questions

1) If  $M = p\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$ ,  $N = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ ,  $O = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ , find the value of  $p$  for which (a)  $M$  and  $N$  are perpendicular to each other (b)  $M$ ,  $N$  and  $O$  are coplanar.

### Solution

$$M = p\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}, N = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}, O = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$

$$\text{a) } \vec{M} \cdot \vec{N} = (p\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}) \cdot (4\mathbf{i} + 3\mathbf{j} - \mathbf{k})$$
$$= 4p - 18 + 3 = 11$$

The perpendicular

$$\vec{M} \cdot \vec{N} = 0$$

$$11 - p = 0$$

$$\therefore p = 11 //$$

$$\text{b) } \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = p(6-3) - 6(8+1) - 3(-12-3)$$
$$= 6p - 3p - 54 - 36 - 9$$
$$= 3p - 54 - 27$$
$$= 3p - 27 = 9 \therefore p = 9 //$$

2) Find the direction cosines and the unit vector along the sum of  $3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ ,  $2\mathbf{i} - \mathbf{j} + 6\mathbf{k}$  and  $5\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ .

### Solution

$$3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$$

The direction cosines are

$$a = 3 \quad b = 2 \quad c = 5$$
$$= \sqrt{3^2 + 2^2 + 5^2}$$

$$= \sqrt{38} = 6.16$$

$$\cos \alpha = \frac{3}{6.16} = 0.481$$

$$\cos \beta = \frac{2}{6.16} = 0.325$$

$$\cos \gamma = \frac{5}{6.16} = 0.812$$

$$2i - j + 6k$$

$$a = 2 \quad b = -1 \quad c = 6$$
$$= \sqrt{2^2 + (-1)^2 + 6^2}$$
$$= \sqrt{41} = 6.40$$

The direction cosines are:

$$\cos \alpha = \frac{2}{6.40} = 0.313$$

$$\cos \beta = \frac{-1}{6.40} = -0.156$$

$$\cos \gamma = \frac{6}{6.40} = 0.938$$

$$5i + 2j - 3k$$

$$a = 5 \quad b = 2 \quad c = -3$$
$$= \sqrt{5^2 + 2^2 + (-3)^2}$$
$$= \sqrt{38} = 6.16$$

The direction cosines are:

$$\cos \alpha = \frac{5}{6.16} = 0.812$$

$$\cos \beta = \frac{2}{6.16} = 0.325$$

$$\cos \gamma = \frac{-3}{6.16} = -0.487$$

The unit vector

$$(3i - 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$$

$$= 10i + 5j + 14k$$

$$= \sqrt{10^2 + 5^2 + 14^2} = 17.92$$

$$= \frac{10i + 5j + 14k}{17.92} \quad \text{or} \quad \frac{10}{17.92}i + \frac{5}{17.92}j + \frac{14}{17.92}k$$