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1) $M = pi - 6j - 3k$

$N = 4i + 3j - k$

$O = i - 3j + 2k$

@ M and N are perpendicular to each other

$M \cdot N = (Pi - 6j - 3k) \cdot (4i + 3j - k)$

$= 4P - 18 + 3$

$= 4P - 15$

Since they are perpendicular to each other

$\cancel{= 4P - 18 + 3}$

$= 4P - 15 = 0$

$\frac{\cancel{4}P}{\cancel{4}} = \frac{15}{4}$

$P = 15/4$

b) M, N and O are Coplanar

$$M \cdot (N \times O) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= P\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$= P(6-(3)) + 6(8-(-1)) - 3(-12-3) = 0$

$= 3P + 6(8+1) - 3(-15) = 0$

$= 3P - 54 + 45 = 0$

$= 3P + 99 = 0$

$= 3P + 99 = 0$

$= \frac{3P}{\cancel{3}} = \frac{-\cancel{9}}{\cancel{3}}$

$P = -3$

2) $\bar{v} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$\bar{v} = 10i + 3j + 8k$

$a_x = 10,\ a_y = 3$ and $a_z = 8$

$|v| = \sqrt{10^2 + 3^2 + 8^2}$

$|v| = \sqrt{100 + 9 + 64}$

$|v| = \sqrt{173} = 13.15$

(i) The direction Cosines are

$\cos\alpha = \dfrac{a_x}{|v|} = \dfrac{10}{13.15} = 0.761$

$\cos\beta = \dfrac{a_y}{|v|} = \dfrac{3}{13.15} = 0.228$

$\cos\gamma = \dfrac{a_z}{|v|} = \dfrac{8}{13.15} = 0.608$

ii) Unit Vector

$e_v = \dfrac{V}{|V|} = \dfrac{10i + 3j + 8k}{13.15}$

3) $f = 3ui + u^2 j + (u+2)k$

$v = 2ui - 3uj + (u-2)k$

$$(F \times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u+2) \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$= i[u^2(u-2) - [-3u(u+2)]] - j[3u(u-2) - 2u(u+2)] + k[-9u^2 - 2u^3]$$

$$= i[u^3 - 2u^2 - [-3u^2 - 6u]] - j[3u^2 - 6u - 2u^2 + 4u] + k[-2u^3 - 9u^2]$$

$$= i[u^3 - 2u^2 + 3u^2 + 6u] - j[u^2 - 10u] + k[-2u^3 - 9u^2]$$

$$= i[u^3 + u^2 + 6u] - j[u^2 - 10u] + k[-2u^3 - 9u^2]$$

$$\int_0^1 (F \times V) = \int i[u^3 + u^2 + 6u] - \int j[u^2 - 10u] + \int k[-2u^3 - 9u^2$$

$$= i\int u^3 + u^2 + 6u - j\int u^2 - 10u + k\int -2u^3 - 9u^2$$

$$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{\cancel{6}^{3}u^2}{\cancel{2}}\right] - j\left[\frac{u^3}{3} - \frac{\cancel{10}^{5}u^2}{\cancel{2}_1}\right] + k\left[-\frac{\cancel{2}^{1}u^4}{\cancel{4}_2} - \frac{\cancel{6}^{3}u^3}{\cancel{2}_1}\right] + C$$

$$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right] - j\left[\frac{u^3}{3} - 5u^2\right] + k\left[-\frac{u^4}{2} - 3u^3\right] + C$$

$$\int_0^1 (F \times V) = i\left[\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2\right] - j\left[\frac{(1)^3}{3} - 5(1)^2\right] + k\left[\frac{-u^4}{2} - 3(1)^3\right] + C$$

$$- \left[0 + C\right]$$

$$\int_0^1 (F \times V) = i\left[\tfrac{1}{4} + \tfrac{1}{3} + 3\right] - j\left[\tfrac{1}{3} - 5\right] + k\left[-\tfrac{1}{2} - 3\right] + \cancel{C} - \cancel{C}$$

$$\int_0^1 (F \times V) = i\left[\tfrac{43}{12}\right] - j\left[-\tfrac{14}{3}\right] + k\left[-\tfrac{7}{2}\right]$$

$$\int_0^1 (F \times V) = \tfrac{43}{12}i + \tfrac{14}{3}j - \tfrac{7}{2}k$$