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18/Eng04/077

Linear Systems EEE 324 Assignment

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LINEAR SYSTEMS EEE 324
ELECTRICAL / ELECTRONICS ENGINEERING
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ASSIGNMENT SOLUTION

(1.) $K_d = 0.03$, $K = 4kN/m$

$$G(s) = \frac{x(s)}{F(s)} = \frac{1/K}{Ts+1}$$

~~G(s)~~

Re arrange into a T.F ; $f(t) \rightarrow \boxed{G(s) = \frac{x(s)}{f(s)}} \rightarrow x(t)$

$$x(s) = \frac{F(s)\, 1/K}{Ts+1}$$

$$F(s) = H/s$$

$$x(s) = \frac{H/K}{s(Ts+1)}$$

$$x(s) = \frac{H/KT}{s(s+1/T)}$$

$$a = 1/T \; ; \; K(1-e^{-at})$$

$$x(s) = H/K\,(1-e^{-t/T})$$

$$x(s) = \text{Unity} = 1$$

$$1 = 4\times10^3 (1 - e^{-t/0.03})$$

$$-0.99975 = -e^{-t/0.03}$$

$$\ln(0.99975) = \frac{-t}{0.03}$$

$$t = 7.5\times10^{-6}\ s$$

$$t \simeq 7.5\mu s$$

$$\frac{\omega}{K_m x} = \frac{1}{Ts+1}$$

$K_m x$ =

(b.) If $F = 100N$

$$x(s) = \frac{F(s)\, 1/K}{Ts+1}$$

2

$$x(s) = \frac{F(s)}{s^2(Ts+1}$$

$$= \frac{F/T}{s^2(s+1/T)}$$

$$= \frac{Fa}{s^2(s+a)}$$

$$x_s = F\{t - kd(1 - e^{-t/T})\}$$

$$x_s = 100\{7.5\times10^{-6} - 0.03(1 - e^{-\frac{7.5\times10^{-6}}{0.03}})\}$$

$$x_s = 16mm$$

(2.) $E_2 = mc\Delta\theta = mc(\theta_2 - \theta_1)$

$$E_1 = mc(\theta_2 - \theta_1)$$

θ = New Temperature

$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta_2 - \theta_1)}{mc(\theta_2 - \theta_1)} = \frac{1}{Ts+1}$$

$$\frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{1}{Ts+1}$$

$$\theta - \theta_2 = \frac{\theta_2 - \theta_1}{Ts+1}$$

Let $\cancel{\theta_2} \; \theta_2 - \theta_1(s) = K(t)$

$$(\theta - \theta_1)(s) = \frac{K(t)}{Ts+1}$$

then

$$(\theta - \theta_1(s)) = \frac{K(t)(1/T)}{s + 1/T}$$

Laplace transform D.f kd

$$= K/s$$

$$(\theta - \theta_1)(s) = K(1/T)$$

$$S(s+1/T)$$

$$LT^{-1}(0-\theta_i) = K(1-e^{-t/T})$$

(3) $\frac{\omega}{K_m x} = \frac{1}{Ts+1}$

$T = 1/K_3 \quad K_m = \frac{K_1 K_2}{K_3}$

$$\omega = \frac{K_m x}{Ts+1}$$

Laplace transform of the step input

$$\omega = \frac{K_m x}{s}\left(\frac{1}{Ts+1}\right)$$

$$\frac{K_m x}{s}\left(\frac{1/T}{s+1/T}\right)$$

$\omega(s) \Rightarrow K_m x(1-e^{-t/T})$

at $t=0$ $K_m x(1-e^{0}) =$ initial

at $t=T$ $K_m x(1-e^{-t/T}) = 0.63\,K_m x$

at $t = 4T = K_m x(1-e^{-4t/T})$

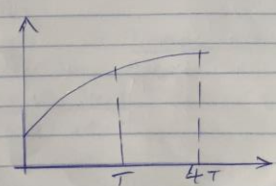
$= 0.981\,K_m x$

for $t=T$

$\Delta\% = (0.632-0)\times 100\% = 63.2\%$

$t = 4T$

$\Delta\% = (0.981-0)\times 100\% = 98.1\%$



(4) $\frac{\theta_o}{\theta_i}(s) = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$

$$\theta_o(s) = \frac{c}{s^2(3s+1)}$$

$$\theta(t) = c(t - 3c(1 - e^{-t/3}))$$

Where $t \neq s$ Log

$$\theta_o(t) = c(t - 3(1))$$

$$\theta_o(t) = ct - 3c$$

$$\theta_t - \theta_2 - \theta_o = ct - (ct - 3c) = 3c$$

$$T = 3 \quad c = 4\,mm/s$$

after 2 seconds

$$\theta = 4 \times 2 = 8mm$$

$\theta_2 = 4mm \times 3 = 12mm$ at steady state

$$\theta_o = 4(2 - 3(1 - e^{2/3}))$$

$$= 2.161mm$$

(5.)(i) $\frac{2}{0.2 + 0.5} = \frac{2/0.5}{0.2s/0.5 + 1}$

$$\rightarrow \frac{4}{0.4s+1}$$

$4 = 0$ gain

0.4 = Time Constant

(ii) $\frac{0.2}{0.05s + 01} = \frac{0.2/0.1}{0.05s/0.1 + 1}$

$$= 2/0.5s + 1$$

2 = DC gain

0.5 = time Constant

(iii) $\frac{2}{3s+1}$ = 2 = DC gain

3 = Time Constant

(iv) $\frac{16}{8s+4} = \frac{16/4}{8/4s + 1} = \frac{4}{2s+1}$

4 = DC gain

2 = Time Constant

(b) $\frac{\omega}{\theta}(s) = \frac{K_m}{\tau_m s + 2}$

$K_m = 15 s^{-1}$

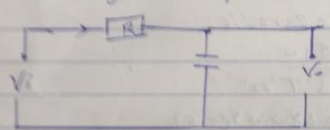
$\tau_m = 4$

$= \frac{15}{4s+2} = \frac{15/2}{4s/2 + 2/2}$

DC gain = 7.5 ms^{-1}

Time Constant = 2 seconds

System Response ?



$T = RC$

$R = 47\Omega \quad C = 20Nf$

$V_i = 5 \sin(2000t)$

$T = 47 \times 20 \times 10^{-6} = 9.4 \times 10^{-9}$

$\left[\frac{V_o}{V_i}\right](\omega) = \frac{1}{Ts+1}$

$G(s) = \frac{1}{Ts+1}$

$G(j\omega) = \frac{1}{9.4\times10^{-9} j\omega + 1} \times \frac{9.4\times10^{-9} j\omega - 1}{9.4\times10^{-9} j\omega - 1}$

$G(\omega) = \frac{9.4\times10^{-9} j\omega - 1}{(9.4\times10^{-9})\omega^2 - 1}$

$G(\omega) = \frac{-1}{(9.4\times10^{-9})\omega^2 - 1}$

Where $\omega = 2000$ rad/s

$\phi = \tan^{-1}\left(\frac{9.4\times10^{-1}(2000)^2 - 1}{1}\right)$

$\theta = -61.99$

$$G(J\omega) = \frac{1}{\sqrt{(9.4\times10^{-5})^2\omega^2+1^2}}$$

$$= \frac{1}{\sqrt{(9.4\times10^{-5})(2000)^2+1^2}}$$

$$= 0.4696$$

$$V_0 = 5\times0.4696 = 2.35$$

(2) $$\frac{X_o}{x_i} = \frac{1}{T^2s^2+2dst+1}$$

$$G(s) = \frac{1}{(1-T^2s^2)+2dTj\omega}$$

$d = 0.2 \quad T = 0.45 \quad \omega = 2s\ rad/s$

$$G(J\omega) = \frac{1-T^2\omega^2-2\delta Tj\omega}{(1-T^2\omega^2)+4\delta^2T^2\omega^2}$$

$$= \frac{1-(0.4)^2(2.5)^2-2(0.2)(0.4)(2.5)}{(1-(0.4)^2(2.5)^2)-4(0.2)^2(0.4)(2.5)^2}$$

$G(j\omega) = 0 = 2.5$

$$\phi = Tan^{-1}\left(\frac{2.5}{0}\right) = 6$$

$Tan^{-1}(\phi) = 90^\circ$

$$G(j\omega) = \sqrt{0^2+2.5^2}$$

$= 2.5$

Amplitude $\Rightarrow 6\times2.5$

$= 15$ //