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1. If $M = Pi + 6j - 3k$, $N = 4i + 3j - k$, $O = i - 3j + 2k$, find the value of P for which (a) M and N are perpendicular to each other (b) M, N, O are coplanar.

solution

a) M and N are perpendicular to each other

$$\vec{M} \cdot \vec{N} = 0$$

$$M \cdot N = (Pi - 6j - 3k) \cdot (4i + 3j - k)$$

$$4P - 18 + 3 = 0$$

$$4P - 15 = 0$$

$$4P = 15$$

$$P = 15/4$$

$$P = 3.75$$

M, N and O are Coplanar

$$M \cdot (N \times O) = 0$$

$$M \cdot (N \times O) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$M \cdot (N \times O) = [6 - 3](P) - [8 + 1](-6) + [-12 - 3](-3)$$

$$3P + 54 + 45 = 0$$

$$3P + 99 = 0$$

$$3P = -99$$

$$P = -33$$

2) find the direction cosines and the unit vector along the sum of $3i+2j+5k$, $2i-j+6k$ and $5i+2j-3k$

Solution

$$3i+2j+5k$$

$$2i-j+6k$$

$$5i+2j-3k$$

$$A = 10i+3j+5k$$

$$|A| = \sqrt{10^2+3^2+5^2} = \sqrt{134} = 11.576$$

a) Direction Cosines

$$\cos \alpha = \frac{x}{|A|}$$

$$\cos \beta = \frac{y}{|A|}$$

$$\cos \gamma = \frac{z}{|A|}$$

$$\cos \alpha = \frac{10}{11.576} = 0.8639$$

$$\cos \beta = \frac{3}{11.576} = 0.2592$$

$$\cos \gamma = \frac{5}{11.576} = 0.4319$$

b) Unit Vector

$$e_A = \frac{\vec{A}}{|A|}$$

$$e_A = \frac{10i+3j+5k}{\sqrt{134}}$$

$$\frac{10}{\sqrt{134}} i + \frac{3}{\sqrt{134}} j + \frac{5}{\sqrt{134}} k$$

3) if $F = 3U^2i + U^2j + (U+2)k$ and $V = 2Ui - 3Uj + (U-2)k$, evaluate the integral of $(F \times V) dU$ from 0 to 1.

solution

$$F \times V = \begin{vmatrix} i & j & k \\ 3U & U^2 & U+2 \\ 2U & -3U & U-2 \end{vmatrix}$$

$$F \times V = [(U^2 \cdot (U-2)) - (-3U)(U+2)]i - [(3U)(U-2) - (2U)(U+2)]j + [(U^3 - 2U^2) - (-3U^2 + 6U)]i - [3U^2 - 6U - (2U^2 + 4U)]j + [U^3 - 2U^2 + 3U^2 + 6U]i - [3U^2 - 6U - 2U^2 - 4U]j + [U^3 + U^2 + 6U]i - (U^2 - 10U)j$$

$$+ [(3U)(-3U) - (2U)(U^2)]k$$

$$(-9U^2 - 2U^3)k$$

$$\int_0^1 F \times V = \int_0^1 [U^3 + U^2 + 6U]i - (U^2 - 10U)j + [-9U^2 - 2U^3]k$$

$$i \left[\frac{U^4}{4} + \frac{U^3}{3} + \frac{6U^2}{2} \right] \Big|_0^1 - \left[\frac{U^3}{3} - \frac{10U^2}{2} \right] \Big|_0^1 + \left[\frac{-9U^3}{3} - \frac{2U^4}{4} \right] \Big|_0^1 k$$

$$\left[\frac{1}{4} + \frac{1}{3} + \frac{3}{1} \right] i - \left[\frac{1}{3} - \frac{5}{1} \right] j + \left[\frac{-3}{1} - \frac{1}{2} \right] k$$

$$\left[\frac{43}{12} \right] i - \left[\frac{-14}{3} \right] j + \left[\frac{-7}{2} \right] k$$

$$\frac{43}{12} i + \frac{14}{3} j - \frac{7}{2} k + C$$