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i) $\frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$

$$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$$

$$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

find the lcm of the right hand side

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$\frac{3x-1}{(x-1)(x-2)(x-2)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

Equate the numerator of the right hand side to that of the left hand side

$3x-1 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$

let $x = 1$

$3(1)-1 = A(1-2)(1-3)+B(1-1)(1-3)+C(1-1)(1-2)$

$2 = A(-1)(-2)+B(0)(-2)+C(0)(-1)$

$\frac{2}{2} = \frac{2A}{2}$ $\quad A = 1$

let $x = 2$

$3(2)-1 = A(2-2)(2-3)+B(2-1)(2-3)+C(2-1)(2-2)$

$5 = A(0)(-1)+B(1)(-1)+C(-1)(0)$

$\frac{5}{-1} = \frac{-B}{-1}$

$B = -5$

let $x = 3$

$3(3)-1 = A(3-2)(3-3)+B(3-1)(3-3)+C(3-1)(3-2)$

$8 = A(1)(0)+B(2)(0)+C(2)(1)$

$\frac{8}{2} = \frac{2C}{2}$ $\quad C = 4$

$\therefore$ The resolve is $\dfrac{(3x-1)}{(x-1)(x-2)(x-3)} = \dfrac{1}{(x-1)} - \dfrac{5}{(x-2)} + \dfrac{4}{(x-3)}$

$$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)}\,dx = \int \frac{1}{(x-1)}\,dx - \int \frac{5}{(x-2)}\,dx + \int \frac{4}{(x-3)}\,dx$$

$$= \ln(x-1) - 5\int \frac{1}{(x-2}\,dx + 4\int \frac{1}{(x-3)}\,dx$$

$$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + c$$

2) $\dfrac{x^2+x+1}{(x+2)(x^2+1)}\,dx$

$$\int \frac{x^2+x+1\,dx}{(x+2)(x^2+1)}$$

Resolve

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$$

find the lcm of rhs

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1) + [(Bx+C)(x+2)]}{(x+2)(x^2+1)}$$

Equate the numerator of rhs to numerator of lhs

$$x^2+x+1 = A(x^2+1) + [(Bx+C)(x+2)]$$

let $x = -2$

$$(-2)^2 + (-2) + 1 = A((-2)^2+1) + [\cancel{Bx+C}\ (B(-2)+C)(-2+2)]$$

$$4 - 2 + 1 = 5A + 0$$

$$\frac{3}{5} = \frac{\cancel{5}A}{\cancel{5}} \qquad A = 3/5$$

From the rhs

$$A(x^2+1) + [(Bx-C)(x+2)]$$

$$= Ax^2 + A + [Bx^2 + 2Bx + Cx + 2C]$$

$$= Ax^2 + A + Bx^2 + Cx + 2C$$

Collect like terms

$$= Ax^2 + Bx^2 + 2Bx + Cx + A + 2C$$

$= x^2(A+B) + x(2B+C) + A + 2C$

$x^2 + x + 1 = x^2(A+B) + x(2B+C) + A + 2C$

Compare Coefficients

$1 = A + B$ —— (i)

$1 = 2B + C$ —— (ii)

$1 = A + 2C$ —— (iii)

Substitute $A = 3/5$ in equation (i)

$1 = 3/5 + B$

$B = 1 - 3/5$

$B = 2/5$

Substitute $B = 2/5$ in equation (ii)

$1 = 2(2/5) + C$

$1 = 4/5 + C$

$C = 1 - 4/5$

$C = 1/5$

$\therefore$ The resolve is $\dfrac{x^2+x+1}{(x+2)(x^2+1)} = \dfrac{3}{5(x+2)} + \dfrac{2x+1}{5(x^2+1)}$

$$\rightarrow \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{3}{5(x+2)}\,dx + \int \frac{2x+1}{5(x^2+1)}\,dx$$

$$= \int \frac{3}{5(x+2)}\,dx + \frac{1}{5}\int \frac{2x+1}{(x^2+1)}\,dx$$

$$= \frac{3}{5}\int \frac{1}{(x+2}\,dx + \frac{1}{5}\left[\int \frac{2x}{(x^2+1}\,dx + \int \frac{1}{(x^2+1)}\,dx\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\left[\frac{2x}{u} \times \frac{1}{2x}\,du + \arctan(x)\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\left[\frac{1}{u}\,du + \arctan(x)\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\left[\ln(x^2+1) + \arctan(x)\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \arctan(x)/5 + C$$

$$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \frac{\arctan(x)}{5} + C$$

3) $\frac{(x^2+1)}{x-3)(x-2)^2} dx$

$$\int \frac{(x^2+1)}{(x-3)(x-2)^2} dx$$

Resolve

$$\frac{(x^2+1)}{(x-3)(x-2)^2} = \frac{(x^2+1)}{(x-3)(x^2-4x+4)}$$

$$\frac{(x^2+1)}{(x-3)(x^2-4x+4)} = \frac{A}{(x-3)} + \frac{Bx+C}{(x^2-4x+4)}$$

find ~~the~~ LCM of the RHS

$$\frac{(x^2+1)}{(x-3)(x^2-4x+4)} = \frac{A(x^2-4x+4)+[(Bx+C)(x-3)]}{(x-3)(x^2-4x+4)}$$

Equate the numerator of the RHS to the numerator ~~of the~~ LHS

$$(x^2+1) = A(x^2-4x+4)+[(Bx+C)(x-3)]$$

Let $x=3$

$$(3)^2+1 = A((3)^2-4(3)+4)+[(B(3)+C)(3-3)]$$

$$10 = A(1)$$

$$A = 10$$

from the RHS

$$A(x^2-4x+4)+[(Bx+C)(x-3)]$$

$$= Ax^2-4Ax+4A+[Bx^2-3Bx+Cx-3C]$$

$$= Ax^2-4Ax+4A+Bx^2-3Bx+Cx-3C$$

Collect like terms

$$= Ax^2+Bx^2-4Ax-3Bx+Cx+4A-3C$$

$$= x^2(A+B)+x(-4A-3B+C)+4A-3C$$

$$(x^2+1) = x^2(A+B)+x(-4A-3B+C)+4A-3C$$

Compare the Coefficient

$1 = A+B$ ---- (i)

$0 = -4A-3B+C$ --- (ii)

$1 = 4A-3C$ ---- (iii)

Substitute $A = 10$ in equation (i)

$1 = A + B$

$1 = 10 + B$

$B = 1 - 10$

$B = -9$

Substitute $B = -9$ and $A = 10$ in equation (ii)

$0 = -4A - 3B + C$

$0 = -4(10) - 3(-9) + C$

$0 = -40 + 27 + C$

$0 = -13 + C$

$C = 13$

$\therefore$ The ~~vector~~ resolve is $\dfrac{(x^2+1)}{(x-3)(x-2)^2} = \dfrac{10}{(x-3)} - \dfrac{9x+13}{(x^2-4x+4)}$

$$\Rightarrow \int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{10}{(x-3)}\,dx - \int \frac{9x+13}{(x^2-4x+4)}\,dx$$

$$= 10\int \frac{1}{(x-3)}\,dx - \int \frac{9x}{(x^2-4x+4)}\,dx + \int \frac{13}{(x^2-4x+4)}\,dx$$

$$= 10\ln(x-3) - 9\ln(x-2) + \frac{18}{(x-2)} - \frac{13}{(x-2)}$$

$$= 10\ln(x-3) - 9\ln(x-2) + \frac{18-13}{(x-2)}$$

$$= 10\ln(x-3) - 9\ln(x-2) + 5/(x-2) + C$$

$$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = 10\ln(x-3) - 9\ln(x-2) + \frac{5}{(x-2)} + C$$

4) $\dfrac{(x^3+x^2+x+1)\,dx}{(x-1)}$

$$\int \frac{(x^3+x^2+x+1)\,dx}{(x-1)} = \int \frac{x^3}{x-1}\,dx + \int \frac{x^2}{x-1}\,dx + \int \frac{x}{(x-1)}\,dx + \int \frac{1}{(x-1)}\,dx$$

$$= \frac{2x^3+3x^2+6x-11}{6} + \ln(x-1) + \frac{x^2}{2} + x + \ln(x-1) + (x-1)$$

$+ \ln(x-1) + \ln(x-1) + c$

$= \dfrac{2x^3 + 3x^2 + 6x - 11}{6} + \dfrac{x^2}{2} + x + x - 1 + \ln(x-1) + \ln(x-1) + \ln(x-1) + \ln(x-1) + c$

$= \dfrac{2x^3 + 3x^2 + 6x - 11 + 3x^2 + 6x + 6x - 6}{6} + 4\ln(x-1) + c$

$= \dfrac{2x^3 + 6x^2 + 18x - 17}{6} + 4\ln(x-1) + c$

$\therefore \displaystyle\int \frac{x^3 + x^2 + x + 1}{(x-1)}\, dx = \frac{2x^3 + 6x^2 + 18x - 17}{6} + 4\ln(x-1) + c$