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Computer science

1.) $N = Pi - 6j - 3k$,
$M = 4i + 3j - k$.
$O = i - 3j + 2k$.

a.) $M \cdot N = (Pi - 6j - 3k) \cdot (4i + 3j - k)$
$= 4p - 18 + 3$
$= 4P - 15$

Since they are perpendicular,
$4p - 15 = 0$.
$\frac{4p}{4} = \frac{15}{4}$

$P = \frac{15}{4}$

b.) M, N and O are coplanar,

2 $M \cdot (N \times O) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$

$= P\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$

$= P(6-3) + 6(8+1) - 3(-12-3)$
$= 3P + 6(9) - 3(-15) = 0$
$= 3p + 54 + 45 = 0$
$= 3p + 99 = 0$.
$\frac{3p}{3} = \frac{-99}{3}$ $\quad P = -33$

2.) $\bar{r} = (5i + 2j + 5k) + (2i - j + 6k) + (3i + 2j - 3k)$,
$\bar{r} = (10i + 3j + 8k)$
$a_x = 10$, $a_y = 3$ and $a_z = 8$.
$|r| = \sqrt{10^2 + 3^2 + 8^2}$
$= \sqrt{173} = 13.15$.

i.) The direction Cosines are:

$\cos\alpha = \frac{a_x}{|v|} = \frac{10}{13.15} = 0.761$

$\cos\beta = \frac{a_y}{|v|} = \frac{3}{13.15} = 0.228$

$\cos\gamma = \frac{a_z}{|v|} = \frac{8}{13.15} = 0.608$

ii.) Unit vector:

$e_v = \frac{v}{|v|} = \frac{10i + 3i + 7k}{13.15}$

3.) $F = 3ui + u^2 j + (u+2)k$

$v = 2ui - 3uj + (u-2)k$

$$(f \times v) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$= i\left[u^2(u-2) - [-3u(u+2)]\right] - j\left[3u(u-2)\right] - 2u(u+2)\Big] + k\left[-9u^2 - 2u^3\right]$$

$$= i\left[u^3 - 2u^2 - \left(-3u^2 - 6u\right)\right] - j\left[3u^2 - 6u - 2u^2 - 4u\right] + k\left[-2u^3 - 9u^2\right]$$

$$= i\left[u^3 - 2u^2 + 3u^2 + 6u\right] - j\left[u^2 - 10u\right] + k\left[-2u^3 - 9u^2\right]$$

$$= i\left[u^3 + u^2 + 6u\right] - j\left[u^2 - 10u\right] + k\left[-2u^3 - 9u^2\right]$$

$$\int_0^1 (f \times v) = \int i\left[u^3 + u^2 + 6u\right] - \int j\left[u^2 - 10u\right] + \int k\left[-2u^3 - 9u^2\right]$$

$$= i\int u^3 + u^2 + 6u - j\int u^2 - 10u + k\int -2u^3 - 9u^2.$$

$$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^2}{2}\right] - j\left[\frac{u^3}{3} - \frac{10u^2}{2}\right] + k\left[\frac{-2u^4}{4} - \frac{9u^3}{3}\right] + c$$

$$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right] + j\left[\frac{u^3}{3} - 5u^2\right] + k\left[-u^4 - 3u^3\right] + c$$

$$= \int_0^1 (f \times v) = i\left[\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2\right] + j\left[\frac{(1)^3}{3} - 5(1)^2\right] + k\left[-(1)^4 - 3(1)^3\right] + c - (0 + c)$$

$$\int_0^1 (f \times v) = i\left[\frac{1}{4} + \frac{1}{3} + 3\right] - j\left[\frac{1}{3} - 5\right] + k\left[-\frac{1}{2} - 3\right] + c - c$$

$$\int_0^1 (f \times v) = i\left[\frac{43}{12}\right] - j\left[-\frac{14}{3}\right] + k\left[-\frac{7}{2}\right]$$

$$\therefore \int_0^1 (f \times v) = \frac{43}{12}i - \frac{14}{3}j + \frac{7}{2}k.$$