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Computer Science

$$1) M = pi - 6j - 3k$$

$$N = 4i + 3j - k$$

$$O = i - 3j + 2k$$

a) M and N are perpendicular

$$\begin{aligned} M \cdot N &= (pi - 6j - 3k) \cdot (4i + 3j - k) \\ &= 4p - 18 + 3 \\ &= 4p - 15 \end{aligned}$$

\therefore Since they are perpendicular

$$4p - 15 = 0$$

$$\frac{4p}{4} = \frac{15}{4}$$

$$p = \frac{15}{4}$$

b) M, N and O are coplanar

$$M \cdot (N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= p \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$$= p(6 - 3) + 6(8 + 1) - 3(-12 - 3)$$

$$= 3p + 6(9) - 3(-15) = 0$$

$$= 3p + 54 + 45 = 0$$

$$= 3p + 99 = 0$$

$$\therefore \frac{3p}{3} = -\frac{99}{3} \quad \therefore p = -33$$

$$2) \vec{v} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j + 4 - 3k)$$

$$\vec{v} = 10i + 3j + 8k$$

$$2x = 10, 2y = 3, 2z = 8$$

$$|V| = \sqrt{10^2 + 3^2 + 8^2}$$

$$= \sqrt{100 + 9 + 64}$$

$$= \sqrt{173} = 13.15$$

i) The direction cosines are

$$\cos \alpha = \frac{2x}{|V|} = \frac{10}{13.15} = 0.761$$

$$\cos \beta = \frac{2y}{|V|} = \frac{3}{13.15} = 0.228$$

$$\cos \gamma = \frac{2z}{|V|} = \frac{8}{13.15} = 0.608$$

ii) Unit vector

$$e_v = \frac{V}{|V|} = \frac{10i + 3j + 8k}{13.15}$$

$$3) F = 3ui + u^2j + (u+2)k$$

$$V = 2ui + 3uj + (u-2)k$$

$$(F \times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & 3u & (u-2) \end{vmatrix} = i \begin{vmatrix} u^2 & (u+2) \\ 3u & (u-2) \end{vmatrix} - j \begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k \begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$= i [u^2(u-2) - [3u(u+2)]] - j [3u(u-2) - 2u(u+2)] + k [-8u^2 - 2u]$$

$$= i [u^3 - 2u^2 - [3u^2 + 6u]] - j [3u^2 - 6u - 2u^2 - 4u] + k [-2u^3 - 9u^2]$$

$$= i [u^3 - 2u^2 + 3u^2 + 6u] - j [u^2 - 10u] + k [-2u^3 - 9u^2]$$

$$= i [u^3 + u^2 + 6u] - j [u^2 - 10u] + k [-2u^3 - 9u^2]$$

$$\int (F \times V) = \int i [u^3 + u^2 + 6u] - \int j [u^2 - 10u] + \int k [-2u^3 - 9u^2]$$

$$= i [u^3 + u^2 + 6u] - j [u^2 - 10u] + k [-2u^3 - 9u^2]$$

$$= \left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^2}{2} \right] - \int \left[\frac{u^3}{3} - \frac{10u^2}{2} \right] + K \left[\frac{-2u^3}{4} - \frac{9u^3}{3} \right] + C$$

$$= \left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2 \right] - \int \left[\frac{u^3}{3} - 5u^2 \right] + K \left[\frac{-u^3}{2} - 3u^3 \right] + C$$

$$\int_0^1 (f(x,v)) = i \left[\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2 \right] - j \left[\frac{(1)^3}{3} - 5(1)^2 \right] + k \left[\frac{-(1)^3}{2} - 3(1)^3 \right] + C$$

$$\int_0^1 (f(x,v)) = i \left[\frac{1}{4} + \frac{1}{3} + 3 \right] - j \left[\frac{1}{3} - 5 \right] + k \left[\frac{-1}{2} - 3 \right] + C$$

$$\int_0^1 (f(x,v)) = i \left[\frac{43}{12} \right] - j \left[\frac{-14}{3} \right] + k \left[\frac{-7}{2} \right]$$

$$\int_0^1 (f(x,v)) = \frac{43}{12} i + \frac{14}{3} j - \frac{7}{2} k$$