19/ENG05/054 MECHATRONICS ENGINEERING

MAT 104

CHIGBU SEAN SEREMI

① $(3x-1)$

$(x-1)(x-2)(x-3)$

$$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)} = \int \frac{A}{(x-1)} + \int \frac{B}{(x-2)} + \int \frac{C}{(x-3)}$$

$$\therefore 3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

When $x=2$,

$$3(2)-1 = B(2-1)(2-3)$$

$$5 = -B$$

$$B = -5$$

When $x=3$,

$$3(3)-1 = C(3-1)(3-2)$$

$$8 = 2C$$

$$C = 4$$

When $x=1$,

$$3(1)-1 = A(1-2)(1-3)$$

$$2 = 2A$$

$$A = 1$$

$$\therefore \int \frac{(3x-1)}{(x-1)(x-2)(x-3)} = \int \frac{1}{(x-1)} + \int \frac{-5}{(x-2)} + \int \frac{4}{(x-3)}$$

$$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$$

2) $\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}\,dx = \int \frac{A}{(x+2)} + \int \frac{Bx+C}{(x^2+1)}$

$\therefore x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

$x^2+x+1 = A(x^2+1) + Bx^2 + 2B + 2C + Cx$

$x^2+x+1 = Ax^2 + Bx^2 + A + 2B + 2C + Cx$

$x^2+x+1 = x^2(A+B) + Cx + (A+2B+2C)$

$A+B=1 \quad -(i)$

$C=1 \quad -(ii)$

$A+2B+2C=1 \quad -(iii)$

put (ii) in (iii)

$A+2B+2=1$

$A+2B=-1$

From (i), $A = 1-B$

$\therefore 1-B+2B=-1$

$B=-2$

$A-2=1$

$A=3$

$\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}\,dx = \int \frac{3\,dx}{(x+2)} + \int \frac{-2\,dx}{(x^2+1)} + \int \frac{1\,dx}{(x^2+1)}$

$= 3\ln(x+2) - 2\ln(x^2+1) + \tan^{-1}x + C$

3) $\int \frac{(x^2+1)}{(x-3)(x-2)^2} = \int \frac{(x^2+1)}{(x-3)(x-2)(x-2)}$

$\int \frac{x^2+1}{(x-3)(x-2)^2} = \int \frac{A}{(x-3)} + \int \frac{B}{(x-2)} + \int \frac{C}{(x-2)^2}$

$[(x-2)^2 = x^2 - 4x + 4]$

$\therefore x^2+1 = A(x-2)(x^2-4x+4) + B(x-3)(x^2-4x+4) + C(x-3)(x-2)$

$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$

$x^2+1 = Ax^2 - 4xA + 4A + Bx^2 - 5xB + 6B + Cx - 3C$

$x^2+1 = (Ax^2 + Bx^2) + (4xA - 5xB + Cx) + (4A + 6B - 3C)$

$x^2+1 = x^2(A+B) + x(-4A-5B+C) + (4A+6B-3C)$

$A + B = 1$ — (i)

$(-4A - 5B + C) = 0$ — (ii)

$(4A + 6B - 3C) = 1$ — (iii)

From (i) $A = 1 - B$ — (iv)

put (iv) into (ii)

$-4(1-B) - 5B + C = 0$

$-4 + 4B - 5B + C = 0 \Rightarrow -B + C - 4 = 0$

$C = B + 4$ — (v)

Put (iv) and (v) in (iii)

$4(1-B) + 6B - 3(B+4) = 1$

$4 - 4B + 6B - 3B - 12 = 1$

$-B - 8 = 1$

$-B = 9$

$B = -9$

$A - 9 = 1 \quad \therefore \quad A = 10$

$4(10) + 6(-9) - 3C = 1$

$C = -5$

$\therefore \int \frac{10}{(x-3)} + \int \frac{-9}{(x-2)} + \int \frac{-5}{(x-2)}$

$10\ln(x-3) - 9\ln(x-2) - 5\ln(x-2) + C$

4) $\int (x^3 + x^2 + x + 1)/(x-1)\, dx$

$x^2 + 2x + 3 + \frac{4}{x-1}$

$x-1 \;\overline{)\; x^3 + x^2 + x + 1}$

$-(x^3 - x^2)$

$2x^2 + x + 1$

$-(2x^2 - 2x)$

$3x + 1$

$-(3x - 3)$

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$\therefore \int x^2 + 2x + 3\, dx + \int \frac{4}{x-1}\, dx$

$= x^3/3 + x^2 + 3x + 4\ln(x-1) + C_{//}$