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1 $M = pi - 6j - 3k$

$N = 4i + 3j - k$

$O = i - 3j + 2k$

(a) M and N are perpendicular to each other

$M \cdot N = (pi - 6j - 3k) \cdot (4i + 3j - k)$

$= 4p - 18 + 3$

$= 4p - 15$

Since they are perpendicular,

$4p - 15 = 0$

$\frac{4p}{4} = \frac{15}{4}$

$p = \frac{15}{4}$

(b) M, N and O are coplanar

$$M \cdot (N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$= p(6-3) + 6(8+1) - 3(-12-3)$

$= 3p + 6(9) - 3(-15) = 0$

$= 3p + 54 + 45 = 0$

$= 3p + 99 = 0$

$\frac{\cancel{3}p}{\cancel{3}} = \frac{-99}{3} \quad p = -33$

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2 $\bar{V} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$\bar{V} = 10i + 3j + 8k$

$a_x = 10$, $a_y = 3$ and $a_z = 8$

$$|V| = \sqrt{10^2 + 3^2 + 8^2}$$
$$= \sqrt{100 + 9 + 64}$$
$$= \sqrt{173} = 13.15$$

i The direction cosines are

$$\cos\alpha = \frac{a_x}{|V|} = \frac{10}{13.15} = 0.761$$

$$\cos\beta = \frac{a_y}{|V|} = \frac{3}{13.15} = 0.228$$

$$\cos\gamma = \frac{a_z}{|V|} = \frac{8}{13.15} = 0.608$$

ii Unit vector

$$e_V = \frac{V}{|V|} = \frac{10i + 3j + 7k}{13.15}$$

3 $F = 3ui + u^2 j + (u+2)k$

$V = 2ui - 3uj + (u-2)k$

$$(F \times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$= i[u^2(u-2) - [-3u(u+2)] - j[3u(u-2)] - 2u(u+2)] + k[-9u^2 - 2u$$

$$= i[u^3 - 2u^2 - [-3u^2 - 6u]] - j[3u^2 - 6u - 2u^2 - 4u] + k[-2u^3 - 9u^2]$$

$$= i[u^3 - 2u^2 + 3u^2 + 6u] - j[u^2 - 10u] + k[-2u^3 - 9u^2]$$

$$= i[u^3 + u^2 + 6u] - j[u^2 - 10u] + k[-2u^3 - 9u^2]$$

$$\int_0^1 (F \times V) = \int i[u^3 + u^2 + 6u] - \int j[u^2 - 10u] + \int k[-2u^3 - 9u^2]$$

$$= i\int u^3 + u^2 + 6u - j\int u^2 - 10u + k\int -2u^3 - 9u^2$$

$$= \left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{\cancel{6}^{3}u^2}{\cancel{2}_{1}}\right] - j\left[\frac{u^3}{3} - \frac{\cancel{10}^{5}u^2}{\cancel{2}_{1}}\right] + k\left[\frac{-\cancel{2}^{1}u^4}{\cancel{4}_{2}} - \frac{\cancel{9}^{3}u^3}{\cancel{3}_{1}}\right] + c$$

$$= \left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right] - j\left[\frac{u^3}{3} - 5u^2\right] + k\left[\frac{-u^4}{2} - 3u^3\right] + c$$

$$\int_0^1 (F \times V) = i\left[\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2\right] - j\left[\frac{(1)^3}{3} - 5(1)^2\right] + k\left[\frac{-(1)^4}{2} - 3(1)^3\right] + c$$

$$- [0 + c]$$

$$\int_0^1 (F \times V) = i\left[\frac{1}{4} + \frac{1}{3} + 3\right] - j\left[\frac{1}{3} - 5\right] + k\left[\frac{-1}{2} - 3\right] + \cancel{c} - \cancel{c}$$

$$\int_0^1 (F \times V) = i\left[\frac{43}{12}\right] - j\left[\frac{-14}{3}\right] + k\left[\frac{-7}{2}\right]$$

$$\int_0^1 (F \times V) = \frac{43}{12} i + \frac{14}{3} j - \frac{7}{2} k$$