19/ENG05/054 MECHATRONICS ENGINEERING

OMOLHABI SEAN SEREMI

MAT 102

(1) $M = pi - 6j - 3k$, $N = 4i + 3j - k$, $O = i - 3j + 2k$

(a) M is perpendicular to N

$\therefore M \cdot N = 1$

$M \cdot N = (p \times 4) + (-6 \times 3) + (-3 \times -1) = 1$

$4p - 18 + 3 = 1$

$4p = 16$

$p = 4$

(b) M, N, and O are coplanar

$[\vec{M}\ \vec{N}\ \vec{O}] = 0$

$$\begin{bmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{bmatrix} = p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$= p(6-3) + 6(8+1) - 3(-12-3) = 0$

$= 3p + 54 + 45 = 0$

$3p + 99 = 0$

$3p = -99$

$\therefore p = -33$

2) sum of vectors $= (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$= (3 + 2 + 5)i + (2 - 1 + 2)j + (5 + 6 - 3)k$

$= 10i + 3j + 8k$

Magnitude $|M| = \sqrt{10^2 + 3^2 + 8^2} = \sqrt{173}$

$\cos\alpha = 10/\sqrt{173}$ $\quad \alpha = \cos^{-1}(10/\sqrt{173}) = 40.51°$

$\cos\beta = 3/\sqrt{173}$ $\quad \beta = \cos^{-1}(3/\sqrt{173}) = 76.82°$

$\cos\gamma = 8/\sqrt{173}$ $\quad \gamma = \cos^{-1}(8/\sqrt{173}) = 52.54°$

Unit Vector $= 10/\sqrt{173}\,i + 3/\sqrt{173}\,j + 8/\sqrt{173}\,k$

3) $F = 3ui + u^2 j + (u+2)k$

$V = 2ui + 3uj + (u-2)k$

$$(F \times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 (u+2) & \\ 2u & 3u (u-2) & \end{vmatrix} = i \begin{vmatrix} u^2 (u+2) & \\ 3u (u-2) & \end{vmatrix} - j \begin{vmatrix} 3u (u+2) & \\ 2u (u-2) & \end{vmatrix} + k \begin{vmatrix} 3u & u^2 \\ 2u & 3u \end{vmatrix}$$

$$= (u^3 - 2u^2 - 3u^2 - 6u)i - (3u^2 - 6u - 2u^2 + 4u)j + (9u^2 - 2u^3)k$$

$$= (u^3 - 5u^2 - 6u)i - (u^2 - 10u)j + (9u^2 - 2u^3)k$$

$$\int_0^1 (u^3 - 5u^2 - 6u)i - (u^2 - 10u)j + (9u^2 - 2u^3)k$$

$$= \left(u^4/4 - 5u^3/3 - 3u^2 \right) i - \left(u^3/3 - 5u^2 \right) j + \left(3u^3 - u^4/2 \right) k \Big|_0^1$$

$$= \left(1^4/4 - 5(1)^3/3 - 3(1)^2 \right) i - \left(1^3/3 - 5(1)^2 \right) j + \left(3(1)^3 - 1^4/2 \right) k$$

$$= -53/12\, i + 14/3\, j + 5/2\, k$$