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Computer Science

(i) $M = Pi - 6j - 3k$
$N = 4i + 3j - k$
$O = i - 3j + 2k$

(a) M and N are perpendicular to each other

$M \cdot N = (Pi - 6j - 3k) \cdot (4i + 3j - k)$
$= 4P - 18 + 3$
$= 4P - 15$

Since they are perpendicular

$4P - 15 = 0$

$\frac{4P}{4} = \frac{15}{4}$ $\quad P = 15/4$

(b) M, N and O are coplanar

$$M \cdot (N \times O) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= P\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$= P(6 - (3)) + 6(8 - (-1)) - 3(-12 - 3) = 0$

$= 3P + 6(8 + 1) - 3(-15) = 0$

$= 3P + 54 + 45 = 0$

$= 3P + 99 = 0$

$\frac{\cancel{3}P}{\cancel{3}} = \frac{-99}{3}$

$P = -11$

(2) $\bar{v} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$\bar{v} = 10i + 3j + 8k$

$a_x = 10$, $a_y = 3$ and $a_z = 8$

$|v| = \sqrt{10^2 + 3^2 + 8^2}$

$|v| = \sqrt{100 + 9 + 64}$

$|v| = \sqrt{173} = 13.15$

The direction cosines are

$\cos \alpha = \frac{a_x}{|v|} = \frac{10}{13.15} = 0.761$

$\cos \beta = \frac{a_y}{|v|} = \frac{3}{13.15} = 0.228$

$\cancel{\cos \alpha}\ \cos \gamma = \frac{a_z}{|v|} = \frac{8}{13.15} = 0.608$

(ii) Unit vector

$e_v = \frac{v}{|v|} = \frac{10i + 3j + 8k}{13.15}$

(3) $F = 3ui + u^2 j + (u+2)k$

$v = 2ui - 3uj + (u-2)k$

$$(F \times v) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i \begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j \begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k \begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$= i[u^2(u-2) - [-3u(u+2)]] - j[3u(u-2) - 2u(u+2)] + k[9$

$= i[u^3 - 2u^2 - 3u^2 - 6u]] - j[3u^2 - 6u - 2u^2 + 4u] + k[-2u^3 - 9u^2$

$= i[u^3 - 2u^2 + 3u^2 + 6u] - j[u^2 - 10u] + k[-2u^3 - 9u^2]$

$= i[u^3 + u^2 + 6u] - j[u^2 - 10j] + k[-2u^3 - 9u^2]$

$$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{\overset{3}{\cancel{6}}u^2}{\cancel{2}}\right] - j\left[\frac{u^3}{3} - \frac{\overset{5}{\cancel{10}}u^2}{\cancel{2}}\right] + k\left[-\frac{\cancel{2}u^4}{\cancel{4}\,2} - \frac{\overset{3}{\cancel{9}}u^3}{\cancel{3}}\right] + c$$

$$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right] - j\left[\frac{u^3}{3} - 5u^2\right] + k\left[-\frac{u^4}{2} - 3u^3\right] + c$$

$$\int_0^1 (f \times v) = i\left[\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2\right] - j\left[\frac{u^3}{3} - 5u^2\right] + k\left[\frac{-(1)^4}{2} - 3(1)^3\right] + c$$

$$- [0 + c]$$

$$\int_0^1 (f \times v) = i\left[\frac{1}{4} + \frac{1}{3} + 3\right] - j\left[\frac{1}{3} - 5\right] + k\left[\frac{-1}{2} - 3\right] + c - c$$

$$\int_0^1 (f \times v) = i\left[\frac{43}{12}\right] - j\left[\frac{-14}{3}\right] + k\left[-\frac{7}{2}\right]$$

$$\int_0^1 (f \times v) = \frac{43}{12}i + \frac{14}{3}j - \frac{7k}{2}$$