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Mat 102

1) $M = pi - 6j - 3k$
$N = 4i + 3j - k$
$O = i - 3j + 2k$

a) M and N are perpendicular
$M.N = (pi - 6j - 3k).(4i + 3j - k)$
$= 4p - 18 + 3$
$= 4p - 15$
Since they are perpendicular,
$4p - 15 = 0$
$\frac{4p}{4} = \frac{15}{4}$
$p = \frac{15}{4}$

b) M, N and O are coplanar.
$$M.(N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$
$= p(6-3) + 6(8+1) - 3(-12-3)$
$= 3p + 6(9) - 3(-15) = 0$
$= 3p + 99 = 0$
$\frac{3p}{3} = \frac{-99}{3}$
$p = -33$

2) $\bar{v} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$\bar{v} = 10i + 3j + 8k$

$a_x = 10$, $a_y = 3$ and $a_z = 8$

$|v| = \sqrt{10^2 + 3^2 + 8^2}$

$= 13.15$

(i) the direction of cosines.

$\cos\alpha = \frac{a_x}{|v|} = \frac{10}{13.15}$

$= 0.761$

$\cos\beta = \frac{a_y}{|v|} = \frac{3}{13.15}$

$= 0.228$

$\cos\gamma = \frac{a_z}{|v|} = \frac{8}{13.15}$

$= 0.608$.

(ii) Unit vector

$e_v = \frac{v}{|v|} = \frac{10i + 3j + 7k}{13.15}$

3) $F = 3ui + u^2 j + (u+2)k$

$v = 2ui - 3uj + (u-2)k$

$$(f \times v) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$= i\left[u^2(u-2) - [-3u(u+2)]\right] - j\left[3u(u-2) - 2u(u+2)\right] + k\left[-9u^2 - 2u\right]$

$= i\left[u^3 - 2u^2 - [-3u^2 - 6u]\right] - j(3u^2 - 6u + 2u^2 - 4u) + k\left[-2u^3 - 9u^2\right]$

$= i(u^3 - 2u^2 + 3u^2 + 6u) - j(u^2 - 10u) + k(-2u^3 - 9u^2)$

$= i(u^3 + u^2 + 6u) - j(u^2 - 10u) + k(-2u^3 - 9u^2)$

$\int_0^1 (f \times v) = \int i(u^3 + u^2 + 6u) - \int j(u^2 - 10u) + \int k(-2u^3 - 9u^2)$

$= i\int u^3 + u^2 + 6u - j\int u^2 - 10u + k\int -2u^3 - 9u^2$

$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^2}{2}\right] - j\left[\frac{u^3}{3} - \frac{10u^2}{2}\right] + k\left[\frac{-2u^4}{4} - \frac{9u^3}{3}\right] + c$

$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right] - j\left[\frac{u^3}{3} - 5u^2\right] + k\left[\frac{-u^4}{2} - 3u^3\right] + c$

$\int_0^1 (f \times v) = i\left[\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2\right] - j\left[\frac{(1)^3}{3} - 5(1)^2\right] + k\left[\frac{-(1)^2}{2} - 3(1)^3\right] + c$

$- (0 + c)$

$\int_0^1 (f \times v) = i\left[\frac{1}{4} + \frac{1}{3} + 3\right] - j\left[\frac{1}{3} - 5\right] + k\left[\frac{-1}{2} - 3\right] + c - c$

$\int_0^1 (f \times v) = i\left[\frac{43}{12}\right] - j\left[\frac{-14}{3}\right] + k\left[\frac{-7}{2}\right]$

$\int_0^1 (f \times v) = \frac{43}{12}i + \frac{14}{3}j - \frac{7}{2}k$