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MAT 104

Find the integral of the following

1) $\dfrac{(3x-1)}{(x-1)(x-2)(x-3)}\,dx$

Solution

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} = \int \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}\,dx$$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

To find A let $x = 1$

$$3x-1 = A(-1)(-2)$$

$$3(1)-1 = 2A$$

$$2 = 2A$$

$$A = 1$$

To find B let $x = 2$

$$3(2)-1 = B(1)(-1)$$

$$5 = -B$$

$$B = -5$$

To find C let $x = 3$

$$3(3)-1 = C(2)(1)$$

$$8 = 2C$$

$$C = 4$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx = \int \frac{1}{x-1}\,dx - \int \frac{5}{x-2}\,dx + \int \frac{4}{x-2}\,dx$$

$$\ln(x-1) - 5\ln(x-2) + 4\ln(x-2) + C$$

2) $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{x^2+x+1}{(x+2)(x+1)^2}\,dx$

Solution

$$\int \frac{x^2+x+1}{x+2\ (x+1)^2}\,dx = \int \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\,dx$$

$$x^2+x+1 = A(x+1)^2 + B(x+2)(x+1) + C(x+2)$$

$$x^2+x+1 = A(x^2+2x+1)\ B(x^2+3x+2) + C(x+2)$$

$$x^2+x+1 = Ax^2+2Ax+A+Bx^2+3Bx+2B+Cx+2C$$

$1 = A+B$ —— (I)

$1 = 2A+3B+C$ —— (II)

$1 = A+2B+2C$ —— (III)

$A = 1-B$ —— (IV)

Taking (II) and (III)

$2 \times\ 2A+3B+C = 1$

$1 \times\ A+2B+2C = 1$

$-\ 4A+6B+2C = 2$

$A+2B+2C = 1$

$3A+4B = 1$ —— (V)

Substitute IV into IV

$3(1-B)+4B = 1$

$3-3B+4B = 1$

$3+B = 1$

$B = -2$

Substitute $B = -2$ into IV

$A = 1-(-2)$

$A = 3$

Substitute $A = 3$, $B = -2$ into (II)

$2(3)+3(-2)+C = 1$

$6-6+C = 1$

$C = 1$

$$\int \frac{x^2+x+1}{(x+2)(x+1)^2} = \int \frac{3}{x+2}\,dx - \int \frac{2}{x+1}\,dx + \int \frac{1}{(x+1)^2}\,dx$$

$$3\ln(x+2) - 2\ln(x+1) - (x+1)^{-1} + C$$

3) $\int \frac{(x^2+1)}{(x-3)(x-2)^2} dx$

Solution

$$\int \frac{x^2+1}{(x-3)(x-2)^2} dx = \int \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} dx$$

$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$

$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$

$x^2 = Ax^2 + Bx^2$

$1 = 4A + 6B - 3C$

$0x = -4Ax - 5Bx + Cx$

$A + B = 1$ —— (i)

$-4A - 5B + C = 0$ —— (ii)

$4A + 6B - 3C = 1$ —— (iii)

Taking (ii) and (iii)

$3 \times \quad -4A - 5B + C = 0$

$1 \times \quad 4A + 6B - 3C = 1$

$-AB - 2C = 1$

$C = 1 - B$

2

$-12A - 15B + 3C = 0$

$+ \quad 4A + 6B - 3C = 1$

$-8A - 9B = 1$ —— v

$A = 1 - B$ —— iv

Substitute iv into v

$-8(1-B) - 9B = 1$

$-8 + 8B - 9B = 1$

$-8 - B = 1$

$-B = 1 + 8$

$B = -9$

Substitute $B = -9$ into IV

$A = 1 - (-9)$

$A = 10$

Substitute $B = -9$, $A = 10$ into II

$-4A + 5B + C = 0$

$-4(10) - 5(-9) + C = 0$

$-40 + 45 + C = 0$

$5 + C = 0$

$C = -5$

$$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{10}{x-3}\,dx - \int \frac{9}{x-2}\,dx - \int \frac{5}{(x-2)^2}\,dx$$

$$10\ln(x-3) - 9\ln(x-2) + 5(x-2)^{-1} + C$$

$$4\int \frac{(x^3+x^2+x+1)}{x-1}\,dx$$

solution

$$\int \frac{x^3+x^2+x+1}{x-1}\,dx$$

$$\begin{array}{r|l} x-1 & x^2+2x+3 \\ \hline & x^3+x^2+x+1 \\ & -x^3-x^2 \\ \hline & 2x^2+x+1 \\ & -2x^2-2x \\ \hline & 3x+1 \\ & -3x-3 \\ \hline & 4 \end{array}$$

$$\int \frac{x^3+x^2+x+1}{x-1}\,dx = \int x^2+2x+8\,dx + \int \frac{4}{x-1}\,dx$$

$$\frac{x^3}{3} + \frac{2x^2}{2} + 8x + 4\ln(x-1) + C$$

$$\frac{x^3}{3} + x^2 + 8x + 4\ln(x-1) + C$$