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# Computer science

1 $m = Pi - 6j - 3k$

$N = 4i + 3j - k$

$O = i - 3j + 2k$

a $M$ & $N$ are perpendicular

$M \cdot N = (Pi - 6j - 3k) \cdot (4i + 3j - k)$

$= 4P - 18 + 3$

$= 4p - 15$

Since they are perpendicular

$4p - 15 = 0$

$\frac{4p}{4} = \frac{15}{4}$

$p = 15/4$

b $M$, $N$ and $O$ are coplanar

$$M \cdot (N \times O) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= P \begin{vmatrix} 3 & 1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & -2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$P(6-3) + 6(8+1) - 3(-12-3)$

$= \cancel{3p + 6}\ p(3) + 6(9) - 3(15)$

$= 3p + 54 + 45 = 0$

$= 3p + 99 = 0$

$\frac{3p}{3} = \frac{-99}{3}$

$p = -33$

2 $\vec{v} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$\vec{v} = 10i + 3j + 8k$

$a_x = 10$, $a_y = 3$ and $a_z = 8$

$|v| = \sqrt{10^2+3^2+8^2}$
$= \sqrt{100+9+64}$
$= \sqrt{173} = 13.15$

The direction of cosines are

$\cos\alpha = \frac{a_x}{|v|} = \frac{10}{13.15} = 0.761$

$\cos\beta = \frac{a_y}{|v|} = \frac{3}{13.15} = 0.228$

Unit vector

$e_v = \frac{v}{|v|} = \frac{10i+3i+7k}{13.10}$

$F = 3ui + u^2j + (u+2)k$
$v = 2vi - 3uj + (u-2)k$

$$(f \times v) = \begin{vmatrix} i & j & k \\ 3u & v^2 & (u+2) \\ 2u & -3v & (v+2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k\begin{vmatrix} 3u & -u^2 \\ 2u & -3v \end{vmatrix}$$

$= i[u^2(u-2) - (-3u(u+2))] - j[3u(u-2)] - 2u(u+2) + k(5u^2-2\ldots$
$= i[u^3-2u^2] - [3u^2-6u]] - j[3u^2-6u-2u^2-4v] + k[-2u^3-9u^2]$
$= i[u^3-2u^2+3u^2+6u] - j[u^2-10u] + k[-2u^3-9u^2]$
$= i[u^3+u^2+6u] - j[u^2+10u] + k[-2u^3-9u^2]$

$\int_0^1 (f \times v) = \int i[u^3+u^2+6u] - \int j[u^2-10u] + \int k[-2u-9u^2]$
$= i\int u^3+u^2+6u - \int u^2-10u + k\int -2u^3-9u^2$
$= \left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^2}{2}\right] - j\left[\frac{u^3}{3} - \frac{10u}{2}\right] + k\left[\frac{-2u}{4} - \frac{9u}{3}\right] + c$

$$\int_0^1 (f \times v) = i\left[\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2\right] - j\left[\frac{(1)^3}{3} - 5(1)^2\right] + k\left[-\frac{(1)^2}{2} - 3(1)^2\right]$$

$$+ c - [0 + c]$$

$$\int_0^1 (f \times v) = i\left[\frac{1}{4} + \frac{1}{3} + 3\right] - j\left[\frac{1}{3} - 5\right] + k\left[-\frac{1}{2} - 3\right] + c - c$$

$$\int_0^1 (f \times v) = i\left[\frac{43}{12}\right] - j\left[-\frac{14}{3}\right] + k\left[-\frac{7}{2}\right]$$

$$\int_0^1 (f \times v) = \frac{43}{12} i + \frac{14}{3} j - \frac{7}{2} k$$