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Mat 102 Assignment

(1) $M = pi - 6j - 3k$
$N = 4i + 3j - k$
$O = i - 3j + 2k$

a.) M and N are perpendicular to each other

$M \cdot N = (pi - 6j - 3k) \cdot (4i + 3j - k)$
$= 4p - 18j + 3$
$= 4p - 15$

Since they are perpendicular
$4p - 15 = 0$
$\frac{4p}{4} = \frac{15}{4}$
$p = 15/4$

(b) M, N and O are coplanar

$M \cdot (N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$

$= p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$

$= p(6-3) + 6(8+1) - 3(-12-3)$
$= 3p + 6(9) - 3(-15) = 0$
$= 3p + 54 + 45 = 0$
$= 3p + 99 = 0$
$\frac{3p}{3} = \frac{-99}{3}$
$p = -33$

(2.) $\bar{V} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$\bar{V} = 10i + 3j + 8k$

$a_x = 10,\ a_y = 3$ and $a_z = 8$

$|V| = \sqrt{10^2 + 3^2 + 8^2}$

$|V| = \sqrt{100 + 9 + 64}$

$= \sqrt{173} = 13.15$

i The direction cosines are

$\cos\alpha = \frac{a_x}{|V|} = \frac{10}{13.15} = 0.761$

$\cos\beta = \frac{a_y}{|V|} = \frac{3}{13.15} = 0.228$

$\cos\gamma = \frac{a_z}{|V|} = \frac{8}{13.15} = 0.608$

ii unit vector

$e_V = \frac{V}{|V|} = \frac{10i + 3j + 8k}{13.18}$

(3.) $f = 3ui + u^2 j + (u+2)k$ and $V = 2ui - 3uj + (u-2)k$

$(f \times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & u+2 \\ 2u & -3u & u-2 \end{vmatrix}$

$= i\begin{vmatrix} u^2 & u+2 \\ -3u & u-2 \end{vmatrix} - j\begin{vmatrix} 3u & u+2 \\ 2u & u-2 \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$

$= i[u^2(u-2) - (-3u(u+2))] - j[3u(u-2) - 2u(u+2)]$

$+ k[-2u^3 - 9u^2]$

$= i[u^3 + u^2 + 6u] - j[u^2 - 10u] + k[-2u^3 - 9u^2]$

$\int_0^1 (f \times V) = \int i[u^3 + u^2 + 6u] - \int j[u^2 - 10u] + \int k[-2u^3 - 9u^2]$

$= \left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^2}{2}\right] - j\left[\frac{u^3}{3} - \frac{10u^2}{2}\right] + k\left[\frac{-8u^4}{4} - \frac{9u^3}{3}\right] + c$

$$= \left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right] - j\left[\frac{u^3}{3} - 5u^2\right] + k\left[\frac{-u^4}{2} - 5u^3\right] + c$$

$$\int_0^1 (f \times v) = i\left[\frac{1}{4} + \frac{1}{3} + 3\right] - j\left[\frac{1}{3} - 5\right] + k\left[\frac{-1}{2} - 5\right] + c - c$$

$$\int_0^1 (f \times v) = i\left[\frac{43}{12}\right] - j\left[\frac{14}{3}\right] + k\left[\frac{-7}{2}\right]$$

$$\int_0^1 (f \times v) = \frac{43}{12} i + \frac{14}{3} j - \frac{7}{2} k$$