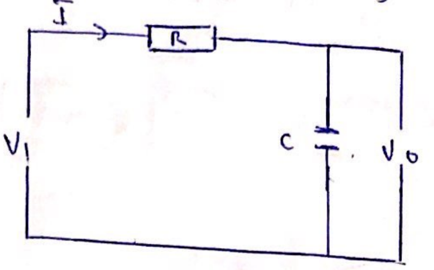
Obot Mkpouto Obot

17/ENG04/046

Electrical and Electronics

Linear systems assignment

1



$$\left(\frac{V_0}{V_1}\right)(s) = \frac{1}{(Ts+1)}$$

where $T = RC$

$R = 47\Omega \quad C = 20\mu F$

$V_1 = 5\sin(2000t)$

$T = 47 \times 20 \times 10^{-6} = 9.4 \times 10^{-4}$

$$G(s) = \frac{1}{(Ts+1)}$$

$$G(j\omega) = \frac{1}{9.4\times10^{-4} j\omega + 1} = \frac{9.4\times10^{-4} j\omega - 1}{9.4\times10^{-4} j\omega - 1}$$

$$G(j\omega) = \frac{9.4\times10^{-4}_{j\omega} - 1}{(9.4\times10^{-4}) j\omega^2 - 1}$$

$$G(j\omega) = \frac{-1}{(9.4\times10^{-4})^2\omega^2 - 1} + \frac{9.4\times10^{-4}}{(9.4\times10^{-4})^2\omega^2 - 1} j\omega$$

where $\omega = 2000\,rad\,s^{-1}$

$$\phi = \tan^{-1}\left[\frac{9.4\times10^{-4}(2000)}{(9.4\times10^{-4})\omega^2 - 1}\right] = \tan^{-1}\left[\frac{9.4\times10^{-4}(2000)}{-1}\right]$$

$\phi = -61.99°$

$$|G(j\omega)| = \frac{1}{\sqrt{(9.4\times10^{-4})^2\omega^2 + 1^2}} \rightarrow \frac{1}{\sqrt{(9.4\times10^{-4})^2(2000)^2 + 1^2}} = 0.46$$

$V_0 =$ amplitude $= 5 \times 0.4196 = 2.348$

$\phi = 2000t - 61.99°$

$v_0 = 2.348 \sin(2000t - 61.99°)$

2) $\frac{x_0}{x_1} = \frac{1}{T^2 s^2 + 2\delta T s + 1} = G(s) = \frac{1}{(1-T^2 s^2) + 2\delta T s}$

$$G(j\omega) = \frac{1}{(1-T^2\omega^2) + 2\delta T j\omega}$$

$\delta = 0.2$ $\quad T = 0.4$ sec $\quad \omega = 2.5$ rad/s

$$G(j\omega) = \frac{1 - T^2\omega^2 - 2\delta T j\omega}{(1-T^2\omega^2)^2 + 4\delta T^2\omega^2}$$

$$\Rightarrow \frac{1-(0.4)^2(2.5)^2 - 2(0.2)(0.4)(2.5)}{(1-(0.4)^2(2.5)^2)^2 + 4(0.2)^2(0.4)^2(2.5)^2}$$

$G(j\omega) = 0 - 2.5j$

$\varphi = \tan^{-1}\left[\frac{2.5}{0}\right] = \tan(\infty) = 90°$

$|G(j\omega)| = \sqrt{0^2 + (2.5)^2}$

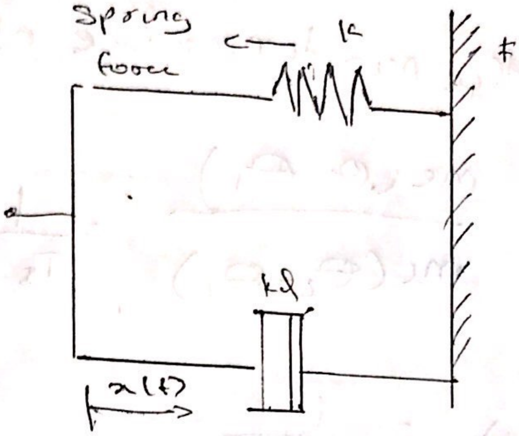
$= 2.5$

amplitude $= 6 \times 2.5$

$= 15$

Obot Mkpouto Obot
17/ENG04/046
Elect / Elect

1)



Spring → $k(x-0)$

F damper ⇒ $kd \frac{d(x-0)}{dt}$

$F(t) \Rightarrow F(t)$

Newton's law ⇒ $F(t) - k(x-0) - kd\frac{d(x-0)}{dt} = 0$

$0 = F(t) - kx - kd\frac{dx}{dt}$ laplace transform ⇒ $F(s) - kx(s) - kds\,x(s)$

$F(s) - kx(s) - kds\; x(s) = 0$

$F(s) = [k + kds]\, x(s)$

$G(s) = \frac{x(s)}{F(s)} = \frac{1}{k+ds} \Rightarrow \frac{1/k}{1+[kd/k]s}$ compare to $\frac{1/k}{Ts+1}$

$\Rightarrow T = \frac{kd}{k} \Rightarrow \frac{0.03}{4\times10^{3}} = 0.75\times10^{-5}$ seconds $\Rightarrow 7.5\times10^{-6}$ s

7.5 μs

3) ε_2 = New energy
ε_1 = initial Energy

$\varepsilon_2 = mc\Delta\theta \Rightarrow \varepsilon_2 = mc(\theta - \theta_1)$

$\varepsilon_1 = mc\Delta\theta = \varepsilon_2 = mc(\theta_2 - \theta_1)$

where θ is the new temperature of the metal

$$G(s) = \frac{\varepsilon_2}{\varepsilon_1} = \frac{mc(\theta - \theta_1)}{mc(\theta_2 - \theta_1)} = \frac{1}{Ts+1}$$

$$= \frac{\theta - \theta_1}{\theta_2 - \theta_1}(s) = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

let $\theta_2 - \theta_1(s) = K(t)$ — (1)

$$(\theta - \theta_1)(s) = \frac{K(t)}{Ts+1}$$

$$(\theta - \theta)_1(s) = \frac{K(t)(1/T)}{s + 1/T}$$

taking the laplace transform of $K(t)$

$= K/s$

$$(\theta - \theta_1)(s) = \frac{k(1/T)}{s(s+1/T)}$$

Inverse lap lase transform

$$(\theta - \theta_1) = k[1 - e^{-t/T}]$$

from (1) $\quad k = \theta_2 - \theta_1$

$$(\theta - \theta_1) = (\theta_2 - \theta_1)[1 - e^{t/T}]$$

3)

k_1	63.2%	44.9%
$k_2 = x$	$t = T$	$t = 4T$
$\delta = k_s$		

$$\frac{w}{km\,2} = \frac{1}{Ts+1}$$

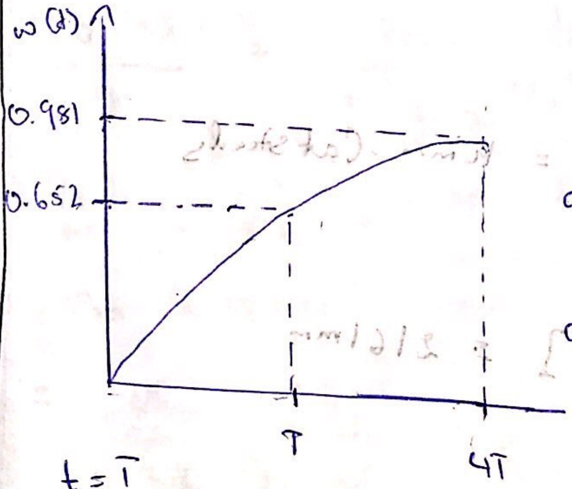
$$T = \frac{1}{ks} \qquad km = \frac{k_1 k_2}{k_s}$$

$$w = \frac{kmx}{Ts+1}$$ taking the laplace transform of step input

$$w = \frac{kmx}{s}\left(\frac{1}{Ts+1}\right)$$

$$\frac{\cancel{kmx}}{\cancel{s}} \Big/ \left(\frac{1/T}{\cancel{s+1/T}}\right) \qquad \frac{kmx}{s}\left(\frac{1/T}{s+1/T}\right)$$

$$w(t) \Rightarrow kmx\,[1 - e^{-t/T}]$$



at $t = 0$ $\quad kmx[1 - e^{0}] = 0$ (initial)

at $t = T$ $\quad kmx\,(1 - e^{-T/T}] = 0.632\,kmx$

$= 0.632\,kmx$

at $t = 4T$ $\quad kmx[1 - e^{-4T/T}] =$

$0.981\,kmx$

$t = T$

$$\Delta\% = [0.632 - 0]\times 100\% = 63.2\%$$

~~fo~~ fo $t = 4T$

$$\Delta\% = [0.981 - 0]\times 100\% = 98.1\%$$

Scanned with CamScanner

4

$\theta_i(t) = ct$

$\theta_i(s) = \frac{c}{s^2}$

$\frac{\theta_o(s)}{\theta_i} = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$

$\theta_o(s) = \frac{c}{s^2(3s+1)}$

$\theta_o(t) = \frac{c/3}{s^2(s+1/3)}$

$\theta_o(t) = c\left[t - 3(1-e^{-t/3})\right]$ —(1)

when t is large

$\theta_o(t) = c[t - 3(1)]$

$\theta_o(t) = ct - 3c$

$\theta_e = \theta_i - \theta_o = ct - [ct - 3c] = 3c$

$T = 3 \qquad c = 4mm/s$

after 2 seconds

$\theta_r = 4 \times 2 = 8mm$

$\theta_e = c \times 3 \Rightarrow 4mm/s \times 3 = 12mm$ - (at steady

θ_o from (1)

$\theta_o = 4\left[2 - 3(1 - e^{-2/3})\right] = 2.161mm$

3(i) $\frac{2}{0.2s+0.5} \rightarrow \frac{2/0.5}{0.2s/0.5+1}$

$\Rightarrow \frac{4}{0.4s+1}$ Compare ~~Compare~~ $\frac{K}{Ts+1}$

4 = D.C gain 0.4 = Time constant

ii $\frac{0.2}{0.05s+0.1} = \frac{0.2/0.1}{0.05s/0.1+0.1/0.1} = \frac{2}{0.5s+1}$

2 = D.C gain 0.5 = Time constant

(iii) $\frac{2}{3s+1} \Rightarrow$ 2 = DC gain
3 = Time constant

iv $\frac{16}{8s+4} \Rightarrow \frac{16/4}{8/4s+1} = \frac{4}{2s+1}$

4 = DC gain 2 = time constant

6) $\frac{\omega(s)}{\theta} = \frac{km}{Tm s+2}$

km = $15s^{-1}$

Tm = 4

$= \cancel{\frac{15}{4s+4}} \quad \frac{15}{4s+2} = \frac{15/2}{4s/2+1} = \frac{7.5}{2s+1}$

Dc gain = $7.5s^{-1}$ Time constant = 2

Scanned with CamScanner