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1) $M = pi - 6j - 3k$

$N = 4i + 3j - k$

$O = i - 3j + 2k$

a) Find the value of p for which M and N are perpendicular to each other.

Solution

$\bar{M} \cdot \bar{N} = 0$

$\bar{M} \cdot \bar{N} = (pi - 6j - 3k) \cdot (4i + 3j - k) = 0$

$= 4p - 18 + 3 = 0$

$= 4p - 15 = 0$

$= \frac{4p}{4} = \frac{15}{4}$

$= 3.75 \approx 4$

b) Find the value of p for which M, N and O are coplanar

$$\bar{M} \cdot (N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} - (-6)\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + (-3)\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$p(6 - 3) + 6(8 - (-1)) - 3(-12 - 3)$

$p(3) + 6(9) - 3(-15)$

$3p + 54 + 45$

$3p + 99$

$\bar{M} \cdot (N \times O) = 3p + 99 = 0$

$= 3p = -99$

$= p = -33$

2. Find the direction cosines and the unit vector along the sum of $3i + 2j + 5k$, $2i - j + 6k$ and $5i + 2j - 3k$

Solution

$$(3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$$

$$10i + 3j + 8k$$

Let $\bar{R} = 10i + 3j + 8k$

$$a_x = 10, \quad a_y = 3; \quad a_z = 8$$

$$|R| = \sqrt{10^2 + 3^2 + 8^2}$$

$$= \sqrt{173} = 13.2$$

$$\cos\alpha = \frac{a_x}{|r|} = \frac{10}{13.2}$$

$$\cos\beta = \frac{a_y}{|r|} = \frac{3}{13.2}$$

$$\cos\gamma = \frac{a_z}{|r|} = \frac{8}{13.2}$$

Unit vector $e_r = \frac{R}{|r|} = \frac{10i + 3j + 8k}{13.2}$

3. $F = 3ui + u^2 j + (u+2)k$

$V = 2ui - 3uj + (u-2)k$

Find $\int_0^1 (\bar{F} \times \bar{V})$

$$\bar{F} \times \bar{V} = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$i\left(u^2(u-2) - (-3u)(u+2)\right) - j\left(3u(u-2) - 2u(u+2)\right) + k\left(-9u^2 - 2u^3\right)$$

$$i\left(u^3 - 2u^2 - (-3u^2 - 6u)\right) - j\left(3u^2 - 6u - 2u^2 + 4u\right) + k\left(-9u^2 - 2u^3\right)$$

$$i\left(u^3 - 2u^2 + 3u^2 + 6u\right) - j\left(3u^2 - 6u - 2u^2 + 4u\right) + k\left(-9u^2 - 2u^3\right)$$

$$i\left(u^3 + u^2 + 6u\right) - j\left(u^2 - 2u\right) + k\left(-9u^2 - 2u^3\right)$$

$$\int_0^1 \left[(u^3 + u^2 + 6u)i - (u^2 - 2u)j + (-9u^2 - 2u^3)k\right]$$

$$= i\left(\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^3}{2}\right) - j\left(\frac{u^3}{3} - \frac{2u^2}{2}\right) + k\left(-\frac{9u^3}{3} - \frac{2u^4}{4}\right)$$

$$= i\left(\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right) - j\left(\frac{u^3}{3} - \frac{u^2}{2}\right) + k\left(-3u^3 - \frac{u^4}{2}\right)$$

$$= i\left(\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right)\Bigg|_0^1 - j\left(\frac{u^3}{3} - \frac{u^2}{2}\right)\Bigg|_0^1 + k\left(-3u^3 - \frac{u^4}{2}\right)\Bigg|_0^1$$

$$= i\left[\left(\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2\right) - \left(\frac{(0)^4}{4} + \frac{(0)^3}{3} + 3(0)^2\right)\right] - j\left[\left(\frac{(1)^3}{3} - \frac{(1)^2}{2}\right) - \left(\frac{(0)^3}{3} - \frac{(0)^2}{2}\right)\right]$$

$$+ k\left[\left(-3(1)^3 - \frac{(1)^4}{2}\right) - \left(3(0)^3 - \frac{(0)^4}{2}\right)\right]$$

$$= i\left(\frac{1}{4} + \frac{1}{3} + 3\right) - j\left(\frac{1}{3} - \frac{1}{2}\right) + k\left(-3 - \frac{1}{2}\right)$$

$$= \left(\tfrac{43}{12}\right)i - \left(-\tfrac{1}{6}\right)j + \left(-\tfrac{7}{2}\right)k$$

$$\therefore \int_0^1 (\bar{F} \times \bar{V}) = \tfrac{43}{12}\, i + \tfrac{1}{6}\, j - \tfrac{7}{2}\, k$$