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MAT102

1. $M = p\hat{i} - 6\hat{j} - 3\hat{k}$ $N = 4\hat{i} + 3\hat{j} - \hat{k}$, $O = \hat{i} - 3\hat{j} + 2\hat{k}$

for perpedicularity $M \cdot N = 0$

$(p\hat{i} - 6\hat{j} - 3\hat{k}) \cdot (4\hat{i} + 3\hat{j} - \hat{k}) = 0$

$4P - 18 + 3 = 0$

$4P = 15$

$\frac{4P}{4} = \frac{15}{4}$

$P = 15/4$ or 3.75

For coplanar vectors $M \cdot (N \times O) = 0$

$$M \cdot (N \times O) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$0 = P(6-3) - \hat{j}(8+1) + \hat{k}(-12-3)$

~~$3P - 9j$~~

$0 = P(3) + 6(9) + (-3)(-15)$

$0 = 3P + 54 + 45$

$0 = 3P + 99$

$3P = -99$

$\frac{3P}{3} = \frac{-99}{3}$

$P = -33$

② $A = (3\hat{i} + 2\hat{j} + 5\hat{k}) + (2\hat{i} - \hat{j} + 6\hat{k}) + (5\hat{i} + 2\hat{j} \cancel{+} 3\hat{k})$

$A = 10\hat{i} + 3\hat{j} + 8\hat{k}$

$|A| = \sqrt{10^2 + 3^2 + 8^2}$

$|A| = \sqrt{173}$

$|A| = 13.15$

$a_x = 10 \qquad a_y = 3 \qquad a_z = 8$

$\cos\alpha = \frac{a_x}{|A|} = \frac{10}{13.15} = 0.76$

$\cos\beta = \frac{a_y}{|A|} = \frac{3}{13.15} = 0.23$

$\cos\gamma = \frac{a_z}{|A|} = \frac{8}{13.15} = 0.61$

$e_A = \frac{A}{|A|}$

$e_A = \frac{10\hat{i} + 3\hat{j} + 8\hat{k}}{13.15}$

③ $F = 3u\,\hat{i} + u^2\,\hat{j} + (u+2)\,\hat{k}$

$V = 2u\,\hat{i} - 3u\,\hat{j} + (u-2)\,\hat{k}$

$$F \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$= \hat{i}\,\cancel{(u^2 - 2u)} \cancel{-\hat{j}}$

$= \hat{i}\,(u^3 + u^2 - 6u) - \hat{j}\,(u^2 - 10u) + \hat{k}\,(-9u^2 - 2u^3)$

$$\int_0^1 (F \times V)\,du = \int_0^1 \left[(u^3 + u^2 - 6u)\,\hat{i} + (u^2 - 10u)\,\hat{j} + (-9u^2 - 2u^3)\,\hat{k} \right]$$

$$= \hat{i}\left(\frac{u^4}{4} + \frac{u^3}{3} - \frac{6u^2}{4^2} \right)\Bigg|_0^1 - \hat{j}\left(\frac{u^3}{3} - \frac{10u^2}{2} \right)\Bigg|_0^1$$

$$+ \hat{k}\left(\frac{-9u^3}{3} - \frac{2u^4}{4} \right)\Bigg|_0^1$$

$$= \hat{i}\left(\frac{1}{4} + \frac{1}{3} - \frac{6}{1} \right) - \hat{j}\left(\frac{1}{3} - \frac{10}{2} \right)$$

$$+ \hat{k}\left(\frac{-9}{3} - \frac{2}{4} \right)$$

$$= \frac{79}{12}\,\hat{i} + \frac{14}{3}\,\hat{j} - \frac{7}{2}\,\hat{k}$$