

$$1.) M = p i - 6 j - 3 k$$

$$N = 4 i + 3 j - k$$

$$O = i - 3 j + 2 k$$

a.) M and N are perpendicular to each other

$$M \cdot N = (p i - 6 j - 3 k) \cdot (4 i + 3 j - k)$$

$$= 4p - 18 + 3$$

$$= 4p - 15$$

Since they are perpendicular

$$4p - 15 = 0$$

$$\frac{4p}{4} = \frac{15}{4}$$

$$p = 15/4$$

b.) M , N and O are coplanar

$$M \cdot (N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= p \begin{vmatrix} 3 & 1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$$= p(6 - 3) + 6(8 + 1) - 3(-12 - 3)$$

$$= 3p + 54 + 45$$

$$= 3p + 99 = 0$$

$$\frac{3p}{3} = \frac{-99}{3}$$

$$p = -33$$

$$2. \vec{V} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$$

$$= 10i + 3j + 8k$$

$$a_1 = 10, a_2 = 3 \text{ and } a_3 = 8$$

$$|\vec{V}| = \sqrt{10^2 + 3^2 + 8^2}$$

$$= \sqrt{100 + 9 + 64}$$

$$= \sqrt{173} = 13.15$$

$$(i) \cos \alpha = a_1/|\vec{V}| = 10/13.15 = 0.761$$

$$\cos \beta = a_2/|\vec{V}| = 3/13.15 = 0.228$$

$$\cos \gamma = a_3/|\vec{V}| = 8/13.15 = 0.608$$

(ii) Unit vector

$$\hat{e}_V = \frac{\vec{V}}{|\vec{V}|} = \frac{10i + 3j + 8k}{13.15}$$

$$3.) F = 3ui^2 + u^2j + (u+2)k$$

$$= 2ui^2 - 3u^2j + (u-2)k$$

$$(\vec{r} \times \vec{v}) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i \begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j \begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k \begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$= i [u^2(u-2) - [-3u(u+2)]] - j [3u(u-2) - 2u(u+2)] + k [2u(-3u) - 6u^2]$$

$$= i [u^3 + u + 6u] - j [u^2 - 10u + 4] + k [-2u^3 - 6u^2]$$

$$= i \left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u}{2} \right] - j \left[\frac{u^3}{3} + \frac{10u}{2} \right] + k \left[-\frac{2u^4}{4} - \frac{9u^3}{3} \right]$$

$$\left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2 \right] - 1 \left[\frac{u^3}{3} - 5u^2 \right] + k \left[\frac{-e^{11}}{2} - 3(11)^3 \right] + C$$

$$S^0 (B \times v) = i \left[7u + 19 + 3 \right] - 3 \left[13 - 5 \right] + k \left[-43 - 3 \right] + C - C$$

$$= i \left[\frac{43}{12} \right] - 3 \left[\frac{-14}{3} \right] + k \left[\frac{-7}{2} \right]$$

$$= \frac{43i}{12} + \frac{14}{3} - \frac{7}{2}k$$