Name : Opoola Daniel Oluwaseyi
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Department : Mechatronics Engineering

1) $\bar{M} = pi - 6j - 3k$, $\bar{N} = 4i + 3j - k$, $\bar{O} = i - 3j + 2k$

a) Find p if M and N are pependicular

solution

$\bar{M} \cdot \bar{N} = (pi - 6j - 3k) \cdot (4i + 3j - k)$

$\bar{M} \cdot \bar{N} = 4p - 18 + 3$

$\bar{M} \cdot \bar{N} = 4p - 15$

for pependicular vectors

$\bar{M} \cdot \bar{N} = 0$

$\therefore \quad 4p - 15 = 0$

$4p = 15$

$p = 15/4 = 3\,3/4$

$p = 3.75$

b) Find p if $\bar{M}$, $\bar{N}$ and $\bar{O}$ are coplanar

solution

$$\bar{M} \cdot [\bar{N} \times \bar{O}] = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} - (-6)\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + (-3)\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$$= p[6 - 3] + 6[8 - (-1)] - 3[-12 - 3]$$

$$= 3p + 54 + 45$$

$\bar{M} \cdot [\bar{N} \times \bar{O}] = 3p + 99$

for Coplanar vectors $\bar{M} \cdot [\bar{N} \times \bar{O}] = 0$

$3p + 99 = 0$

b) cont.

$$3p + 99 = 0$$

$$3p = -99$$

$$p = -99/3$$

$$\underline{\underline{p = -33}}$$

2) Find the direction cosine and the unit vector along the sum of $3i + 2j + 5k$, $2i - j + 6k$ and $5i + 2j - 3k$

Soln

$(3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$= 10i + 3j + 8k$

Let the sum of vectors be A

$\bar{A} = 10i + 3j + 8k$

Unit vector of A $e_A = \frac{\bar{A}}{|A|}$

$|A| = \sqrt{(10)^2 + (3)^2 + (+8)^2} = \sqrt{100 + 9 + 64}$

$= \sqrt{173} = 13.15$

$e_A = \frac{10i + 3j + 8k}{13.15} = \frac{10}{13.15} i + \frac{3}{13.15} j + \frac{8}{13.15} k$

direction cosine $\Rightarrow$ $\cos\alpha = \frac{a_x}{|A|}$ $\cos\beta = \frac{a_y}{|A|}$ $\cos\gamma = \frac{a_z}{|A|}$

$\cos\alpha = \frac{10i}{13.5}$

$\cos\beta = \frac{3}{13.5} j$

$\cos\gamma = \frac{8}{13.5} k$

3.) If $F = 3ui + u^2 j + (u+2)k$ and $V = 2ui - 3uj + (u-2)k$, evaluate the integral of $(F \times V)\,du$ from 0 to 1

Soln

$$(F \times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & u+2 \\ -3u & u-2 \end{vmatrix} - j\begin{vmatrix} 3u & u+2 \\ 2u & u-2 \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$= i[(u^3 - 2u^2) - (-3u^2 - 6u)] - j[(3u^2 - 6u) - (2u^2 + 4u)] + k[-9u^2 - 2u^3]$$

$$= i[u^3 - 2u^2 + 3u^2 + 6u] - j[3u^2 - 6u - 2u^2 - 4u] + k[-9u^2 - 2u^3]$$

$$\int_0^1 (F \times V)\,du = \int_0^1 i[u^3 + u^2 + 6u] - j[u^2 - 10u] + k[-9u^2 - 2u^3]$$

$$\int (F \times V)\,du = \left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right] i \Big|_0^1 - \left[\frac{u^3}{3} - 5u^2\right] j \Big|_0^1 + \left[-3u^3 - \frac{u^4}{2}\right] k \Big|_0^1$$

when u = 1

$$= \left[\frac{1^4}{4} + \frac{1^3}{3} + 3(1)^2\right] i - \left[\frac{1^3}{3} - 5(1)^2\right] j + \left[-3(1)^3 - \frac{1^4}{2}\right] k$$

$$= \left(\frac{1}{4} + \frac{1}{3} + 3\right) i - \left[\frac{1}{3} - 5\right] j + \left(-3 - \frac{1}{2}\right) k$$

$= 43/12\, i + 14/3\, j - 7/2\, k$ multiply through by 12

$= 43i + 56j - 42k$