Name: Isaac Grace Akowoicho
Department: Petroleum Engineering
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1. If $M = pi - 6j - 3k$, $N = 4i + 3j - k$, $O = i - 3j + 2k$, find the value of p for which (a) M and N are perpendicular to each other (b) M, N and O are coplanar

Solution

a. $\vec{M} \cdot \vec{N} = (pi - 6j - 3k) \cdot (4i + 3j - k)$
$= 4p - 18 + 3$
$\vec{M} \cdot \vec{N} = 4p - 15$

$\vec{M}$ and $\vec{N}$ are perpendicular if $\vec{M} \cdot \vec{N} = 0$

$\vec{M} \cdot \vec{N} = 4p - 15 = 0$
$4p = 15$
$p = 15/4$

b. $$\vec{M} \cdot (\vec{N} \times \vec{O}) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= p \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$= p(6 - 3) + 6(8 + 1) - 3(-12 - 3)$
$= 3p + 54 + 45$
$\vec{M} \cdot (\vec{N} \times \vec{O}) = 3p + 99$

$\vec{M}$, $\vec{N}$ and $\vec{O}$ are coplanar if $\vec{M} \cdot (\vec{N} \times \vec{O}) = 0$

$3p + 99 = 0$
$3p = -99$
$p = -33$

2. Find the direction cosines and the unit vector along the sum of $3i+2j+5k$, $2i-j+6k$ and $5i+2j-3k$.

Solution

let $\vec{A} = 3i+2j+5k$

$\vec{B} = 2i-j+6k$

$\vec{C} = 5i+2j-3k$

$\vec{A}+\vec{B}+\vec{C} = (3i+2j+5k)+(2i-j+6k)+(5i+2j-3k)$

$\vec{A}+\vec{B}+\vec{C} = 10i+3j+8k$

$a_x = 10, \quad a_y = 3, \quad a_z = 8$

$|A+B+C| = \sqrt{10^2+3^2+8^2}$

$= \sqrt{173}$

∴ The direction cosines are

$\cos\alpha = \dfrac{a_x}{|A+B+C|} = \dfrac{10}{\sqrt{173}} = 0.7603$

$\cos\beta = \dfrac{a_y}{|A+B+C|} = \dfrac{3}{\sqrt{173}} = 0.2281$

$\cos\gamma = \dfrac{a_z}{|A+B+C|} = \dfrac{8}{\sqrt{173}} = 0.6082$

The unit vector is

$e_{(A+B+C)} = \dfrac{\vec{A}+\vec{B}+\vec{C}}{|A+B+C|}$

$= \dfrac{10i+3j+8k}{\sqrt{173}}$

$e_{(A+B+C)} = \dfrac{\sqrt{173}\,(10i+3j+8k)}{173}$

3. If $\vec{F} = 3ui + u^2 j + (u+2)k$ and $\vec{V} = 2ui - 3uj + (u-2)k$ evaluate the integral of $(\vec{F} \times \vec{V})\,du$ from 0 to 1.

Solution

$$\vec{F} \times \vec{V} = \begin{vmatrix} i & j & k \\ 3u & u^2 & u+2 \\ 2u & -3u & u-2 \end{vmatrix}$$

$$= i \begin{vmatrix} u^2 & u+2 \\ -3u & u-2 \end{vmatrix} - j \begin{vmatrix} 3u & u+2 \\ 2u & u-2 \end{vmatrix} + k \begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$\cancel{= i[u^3}$$

$$= i\left[(u^3 - 2u^2) + (3u^2 + 6u)\right] - j\left[(3u^2 - 6u) - (2u^2 + 4u)\right] + k\left[-9u^2 - 2u^3\right]$$

$$= (u^3 + u^2 + 6u)i - (u^2 - 10u)j \cancel{+ k} - (9u^2 + 2u^3)k$$

$$\int_0^1 (\vec{F} \times \vec{V})\,du$$

$$= \int_0^1 (u^3 + u^2 + 6u)i\,du - \int_0^1 (u^2 - 10u)j\,du - \int_0^1 (9u^2 + 2u^3)k\,du$$

$$= i\left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right]_0^1 - j\left[\frac{u^3}{3} - 5u^2\right]_0^1 - k\left[\frac{u^4}{2} + 3u^3\right]_0^1$$

$$= i\left[\frac{1}{4} + \frac{1}{3} + 3\right] - j\left[\frac{1}{3} - 5\right] - k\left[\frac{1}{2} + 3\right]$$

$$= \frac{43}{12}i + \frac{14}{3}j - \frac{7}{2}k$$

$$\therefore \int_0^1 (\vec{F} \times \vec{V})\,du = \frac{43}{12}i + \frac{14}{3}j - \frac{7}{2}k$$