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1. $\dfrac{3x-1}{(x-1)(x-2)(x-3)}dx$

Solution

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$\Rightarrow \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$$\Rightarrow 3x-1 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$$

when $x=1$,

$3(1)-1 = A(-1)(-2)+B(0)(-2)+C(0)(-1)$

$2 = 2A$

$\therefore A = 1$

when $x=2$,

$3(2)-1 = A(0)(-1)+B(1)(-1)+C(1)(0)$

$5 = -B$

$\therefore B = -5$

when $x=3$

$3(3)-1 = A(1)(0)+B(2)(0)+C(2)(1)$

$8 = 2C$

$\therefore C = 4$

Hence,

$$\int\frac{3x-1}{(x-1)(x-2)(x-3)}dx = \int\frac{1}{x-1}dx - \int\frac{5}{x-2}dx + \int\frac{4}{x-3}dx$$

$$\therefore \int\frac{3x-1}{(x-1)(x-2)(x-3)}dx = \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$$

2. $\dfrac{x^2+x+1}{(x+2)(x^2+1)}\,dx$

Solution

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$$

$$\Rightarrow x^2+x+1 = Ax^2+A+Bx^2+2Bx+Cx+2C$$

$$x^2+x+1 = x^2(A+B)+x(2B+C)+A+2C$$

Comparing coefficients

$$A+B=1 \text{ ——— } (i)$$

$$2B+C=1 \text{ ——— } (ii)$$

$$A+2C=1 \text{ ——— } (iii)$$

From (i), $A=1-B$ ——— (iv)

Put (iv) in (iii)

$$1-B+2C=1$$

$$B=2C \text{ ——— } (v)$$

Put (v) in (ii)

$$4C+C=1$$

$$5C=1$$

$\therefore\ C=1/5$ ——— (vi)

Put (vi) in (iii)

$$A+2/5=1$$

$\therefore\ A=3/5$ ——— (vii)

Put (vii) in (i)

$$3/5+B=1$$

$\therefore\ B=2/5$

Hence,

$$\int\frac{x^2+x+1}{(x+2)(x^2+1)} = \int\frac{3}{5(x+2)}\,dx+\int\frac{2x}{5(x^2+1)}\,dx+\int\frac{dx}{5(x^2+1)}$$

$$= \frac{3}{5}\ln(x+2)+\frac{1}{5}\ln(x^2+1)+\frac{1}{5}\tan^{-1}x + C$$

3. $\dfrac{x^2+1}{(x-3)(x-2)^2}\,dx$

Solution

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B(x-3)(x-2) + C(x-3)}{(x-3)(x-2)^2}$$

$$\Rightarrow x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3) \quad (*)$$

when $x = 2$,

$$4+1 = A(0)^2 + B(-1)(0) + C(-1)$$

$$5 = -C$$

$$\therefore C = -5$$

when $x = 3$,

$$9+1 = A(1)^2 + B(0)(1) + C(0)$$

$$10 = A$$

$$\therefore A = 10$$

Put $C=-5$ and $A=10$ into $(*)$

$$x^2+1 = 10(x-2)^2 + B(x-3)(x-2) + (-5)(x-3)$$

$$x^2+1 = 10x^2 - 40x + 40 + Bx^2 - 5Bx + 6B - 5x + 15$$

$$Bx^2 - 5Bx + 6B = -9x^2 + 45x - 54$$

$$B(x^2 - 5x + 6) = -9(x^2 - 5x + 6)$$

$$\Rightarrow B = -9$$

Hence,

$$\int \frac{x^2+1}{(x-3)(x-2)^2} = \int \frac{10}{x-3} - \int \frac{9}{x-2} - \int \frac{5}{(x-2)^2}$$

$$= 10\ln(x-3) - 9\ln(x-2) - \frac{5}{x-2} + C$$

4. $\dfrac{x^3+x^2+x+1}{x-1}\,dx$

Solution

$$\begin{array}{r|l} & x^2+2x+3 \\ \hline x-1 & x^3+x^2+x+1 \\ & -(x^3-x^2) \\ & \quad 2x^2+x \\ & \quad -(2x^2-2x) \\ & \qquad 3x+1 \\ & \qquad -(3x-3) \\ & \qquad\quad 4 \end{array}$$

$$\Rightarrow \int \frac{x^3+x^2+x+1}{x-1}\,dx = \int x^2\,dx + \int 2x\,dx + \int 3\,dx + \int \frac{4}{x-1}\,dx$$

$$= \frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + C$$