NAME: TEDEBE MARILYN-ROSE OMOSI
DEPARTMENT: COMPUTER SCIENCE
MATRIC NO: 19/SCI01/086
~~DATE~~ COURSE: MAT 104

1. Find the limit of the function $(4x^2 - \sin x / x^3)$ as $x \to 0$

Soln

$$\lim_{x \to 0} \left\{ \frac{4x^2 - \sin x}{x^3} \right\}$$

By L'hopital's rule, we have

$$\lim_{x \to 0} \left\{ \frac{8x - \cos x}{3x^2} \right\}$$

For the second derivative we have

$$\lim_{x \to 0} \left\{ \frac{8 + \sin x}{6x} \right\}$$

For the third derivative

$$\lim_{x \to 0} \left\{ \frac{\cos x}{6} \right\}$$

$$= \frac{\cos(0)}{6} = 1/6$$

2. If $y = (7x^2 \cos 8x) / e^{3x}$, find the derivative of y with respect to x.

Soln

$$y = \frac{7x^2 \cos 8x}{e^{3x}}$$

$u = 7x^2 \qquad v = \cos 8x \qquad w = e^{3x}$

$$\frac{du}{dx} = 14x, \quad \frac{dv}{dx} = -8\sin 8x, \quad \frac{dw}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = y\left[\frac{1}{u}\cdot\frac{du}{dx} + \frac{1}{v}\cdot\frac{dv}{dx} - \frac{1}{w}\cdot\frac{dw}{dx}\right]$$

$$= \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{1}{7x^2}(14x) + \frac{1}{\cos 8x}(-8\sin 8x) - \frac{1}{e^{3x}}(3e^{3x})\right]$$

$$\frac{dy}{dx} = \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{2}{x} - 8\tan 8x - 3\right]$$

3. If $y = \cos(5x^2 + 6x)$. Find $\frac{dy}{dx}$

Soln

$$y = \cos(5x^2 + 6x)$$

$$u = 5x^2 + 6x, \quad \frac{du}{dx} = 10x + 6$$

$$y = \cos u \quad \frac{dy}{du} = -\sin u$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= (10x + 6) \times -\sin u$$

$$= -(10x + 6)\sin u$$

Recall, $u = (5x^2 + 6x)$

$$= -(10x + 6)\sin(5x^2 + 6x)$$

4. Find the integral of the following

a) $\int \frac{3}{\sqrt{4x+1}}\,dx = 3\int \frac{dx}{4x+1}$

$$u = 4x + 1$$

$$\frac{du}{dx} = 4$$

$dx = \frac{du}{4}$

$3\int \frac{dx}{4x+1} = 3\int \frac{1}{u} \cdot \frac{du}{4} = \frac{3}{4}\int \frac{1}{u}\, du$

$= \frac{3}{4} \ln u + c$ Recall $u = 4x+1$

$= \frac{3\ln(4x+1)}{4} + c$

b) $\int \frac{dx}{x^2+49} = \int \frac{1}{x^2+49}\, dx = \int \frac{1}{49\left(\frac{x^2}{49}+1\right)}\, dx$

$u = \frac{x}{7}, \quad du = \frac{1}{7}\, dx$

$7\, du = dx$

$\frac{1}{49}\int \frac{1}{\left(\frac{x}{7}\right)^2+1}\, dx$

$\frac{1}{49}\int \frac{1}{u^2+1}\, 7\, du$

$\frac{1}{49} \cdot \frac{7}{1}\int \frac{1}{u^2+1}\, du$

$\frac{1}{7}\int \frac{1}{u^2+1}\, du$

$= \frac{1}{7}\tan^{-1}\frac{x}{7} + c$

c) $\int \left(e^{6x} + 9x^3 - \sin 7x + \cos 8x\right) dx$

$\frac{1}{6}e^{6x} + \frac{9x^{3+1}}{3+1} - \left(\frac{1}{7}\cos 7x\right) + \frac{1}{8}\sin 8x + c$

$\frac{1}{6}e^{6x} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C$

Scanned with CamScanner

d) $\int x\sqrt{9+x^2}\,dx$

$u = 9+x^2 \qquad \frac{du}{dx} = 2x, \qquad dx = \frac{du}{2x}$

$\int x\sqrt{u}\,\frac{du}{2x}$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right] + C$

$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}\left[u^{3/2}\right] + C$

$= \frac{1}{3}\left[u^{3/2}\right] + C$

Recall $u = 9+x^2$

$= \frac{1}{3}\left[(9+x^2)^{3/2}\right] + C$

$= \frac{(9+x^2)^{3/2}}{3} + C$ //