

1)  $M = pi - 6j - 3k$ ;  $N = 4i + 3j - k$ ,  $O = i - 3j + 2k$

a)  $M \cdot N \neq 0$  (perpendicular to each other)

$$(pi - 6j - 3k) \cdot (4i + 3j - k) = 0$$

$$4p - 18 + 3 = 0$$

$$4p = 15$$

$$p = \frac{15}{4}$$

b)  $M \cdot (P \times O) = 0$ , (coplanar)

$$M(P \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$$p(6 - 3) + 6(8 + 1) - 3(-12 - 3) = 0$$

$$p(3) + 6(9) - 3(-15) = 0$$

$$3p + 54 + 45 = 0$$

$$3p = -99$$

$$p = -33$$

2)  $U = 3i + 2j + 5k + 2i - j + 6k + 5i + 2j - 3k$

$$U = 10i + 3j + 8k$$

$$|U| = \sqrt{10^2 + 3^2 + 8^2} = \sqrt{173}$$

$$|U| = 13.15$$

$$\cos \alpha = \frac{a_x}{|v|} = \frac{10}{13.15}$$

$$\cos \beta = \frac{a_y}{|v|} = \frac{3}{13.15}$$

$$\cos \gamma = \frac{a_z}{|v|} = \frac{8}{13.15}$$

$$e_v = \frac{v}{|v|} = \frac{10i + 3j + 8k}{13.15}$$

$$b) F = 3v^2 i + v^2 j + (v+2)k, \quad v = 2u i + 3u j + (u-2)k$$

$$\int_0^1 (F \times v) du =$$

$$F \times v = \begin{vmatrix} i & j & k \\ 3v & v^2 & (v+2) \\ 2v & -3v & (v-2) \end{vmatrix}$$

$$= i(v^2(v-2) + 3v(v+2)) - j(3v(v-2) - 2v(v+2)) + k(-9v^2 - 2v^3)$$

$$= i(v^3 - 2v^2 + 3v^2 + 6v) - j(3v^2 - 6v - 2v^2 - 4v) + k(-9v^2 - 2v^3)$$

$$= i(v^3 + v^2 + 6v) - j(v^2 - 10v) + k(-9v^2 - 2v^3)$$

$$= \int_0^1 (i(v^3 + v^2 + 6v) - j(v^2 - 10v) + k(-9v^2 - 2v^3)) du$$

$$= \left[ \left( \frac{v^4}{4} + \frac{v^3}{3} + 3v^2 \right) i - \left( \frac{v^3}{3} - 5v^2 \right) j + \left( -3v^3 - \frac{v^4}{2} \right) k \right]_0^1$$

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$$= \left( \frac{1}{4} + \frac{1}{3} + 3 \right) i - \left( \frac{1}{3} - 5 \right) j + \left( -3 - \frac{1}{2} \right) k$$

$$= \frac{43}{12} i + \frac{14}{3} j - \frac{7}{2} k$$

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