IKPEAMA JOHN
17/ENG04/031
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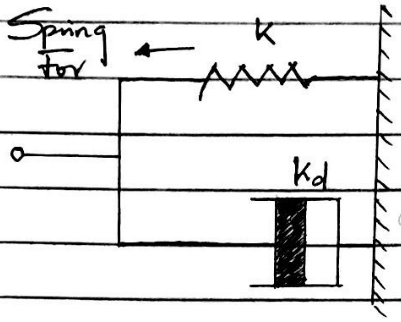
SYSTEM RESPONSE II

1.



Spring $\rightarrow k(x-0)$
F damper $\rightarrow k_d \frac{d(x-0)}{dt}$

$F(t) \rightarrow F(t)$

Newton's law $\rightarrow F(t) - k(x-0) - k_d \frac{d(x-0)}{dt} = 0$

$$0 = F(t) - kx - k_d \frac{dx}{dt} \rightarrow F(s) - kx(s) - k_d s\, x(s)$$

$$F(s) - kx(s) - k_d s\, x(s) = 0$$
$$F(s) = [k + k_d s]\, x(s)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{k + k_d s} = \frac{1/k}{1 + [k_d/k]s} \rightarrow \frac{1/k}{Ts+1}$$

$$\Rightarrow T = \frac{k_d}{k} \Rightarrow \frac{0.03}{4 \times 10^3} = 0.75 \times 10^{-5}\,s$$
$$= 7.5\,\mu s.$$

2i Let's assume E_2 - Final Energy
E_1 - initial Energy.

$$E_2 = mc\Delta\theta \rightarrow mc(\theta - \theta_1)$$
$$E_1 = mc\Delta\theta \rightarrow mc(\theta_2 - \theta_1)$$

θ is the new temperature of the metal.

$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta - \theta_1)}{mc(\theta_2 - \theta_1)} = \frac{1}{Ts+1}$$

$$= \frac{\theta - \theta_1}{\theta_2 - \theta_1}(s) = \frac{1}{\tau s + 1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{\tau s + 1}$$

Let $\theta_2 - \theta_1 (s) = k(t)$

$$(\theta - \theta_1)(s) = \frac{k(t)}{\tau s + 1}$$

$$(\theta - \theta)(s) = \frac{k(t)\left(1/\tau\right)}{s + 1/\tau}$$

$$= \frac{k}{s}$$

$\therefore \frac{k}{s}$ to laplace Form $= k(t)$.

2ii $$(\theta - \theta_1)(s) = \frac{k\left(1/\tau\right)}{s\left(s + 1/\tau\right)}$$

inverse laplace transform.

$$(\theta - \theta_1) = k\left(1 - e^{-t/\tau}\right)$$

From previous solution

$$k = \theta_2 - \theta_1$$

$$(\theta - \theta_1) = (\theta_2 - \theta_1)\left[1 - e^{-t/\tau}\right].$$

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3

$$\frac{\omega}{k_m X} = \frac{1}{Tp+1}$$

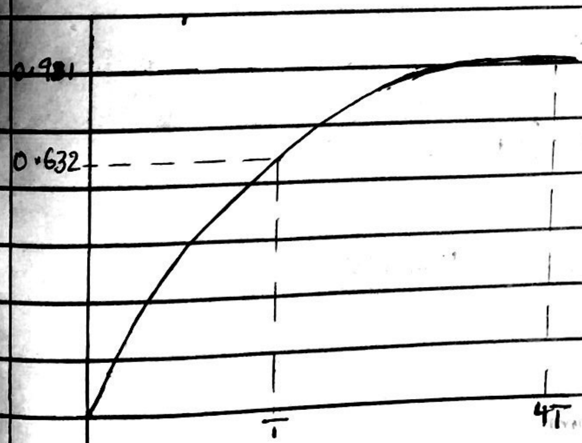
$$T = \frac{1}{K_p} \qquad k_m = \frac{K_1 K_2}{K_3}$$

$$\omega = \frac{k_m x}{Tp+1}$$

$$\omega = \frac{k_m X}{p}\left(\frac{1}{Tp+1}\right)$$

$$\frac{k_m X}{p}\left(\frac{1/T}{p+1/T}\right)$$

$$\omega(t) - k_m X\left[1 - e^{-t/T}\right]$$



at $t=0$ $k_m x(1-e^{0}) = 0$

at $t=T$ $k_m X(1-e^{-1T/T}) = 0.632\,k_m X$

at $t=4T$ $k_m x(1-e^{-4T/T}) = 0.981\,k_m X$

For $t = T$

$$\Delta\% = [0.632 - 0] \times 100\% = 63.2\%$$

For $t = 4T$

$$\Delta\% = [0.981 - 0] \times 100\% = 98.1\%$$

4 $\theta_i(t) = Ct$

$\theta_i(s) = \frac{C}{s^2}$

$\frac{\theta_o}{\theta_i}(s) = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$

$\theta_o(s) = \frac{C}{s^2(3s+1)}$

$\theta_o(s) = \frac{C/3}{s^2(s+1/3)}$

$\theta_o(t) = C[(t-3)][1-e^{-t/3}]$ — (i)

When t is large

$\theta_o(t) = C[t-3(1)]$

$\theta_o(t) = Ct - 3C.$

$\theta_e = \theta_i - \theta_o = Ct - (Ct - 3C) = 3C$

When $T = 3$, $C = 4mm/s.$

after 2 seconds.

$\theta_i = 4 \times 2 = 8mm$

$\theta_e = C \times 3 = 4 \times 3 = 12mm$

θ_o from (i)

$\theta_o = 4[2 - 3(1 - e^{-2/3})] = 2.161mm.$

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SYSTEM RESPONSE I

5. $\frac{2}{0.2 + 0.5} = \frac{2/0.5}{0.2s/0.5 + 1}$

$\downarrow$

$\frac{4}{0.4s + 1} = \frac{k}{\tau s + 1}$

4 = D.C gain 0.4 = Time Constant.

ii $\frac{0.2}{0.05s + 0.1} = \frac{0.2/0.1}{0.05s/0.1 + 0.1/0.1}$

2 = D.C gain 0.5 = Time Constant.

iii $\frac{2}{3s+1}$ = 2 = D.C gain
3 = Time Constant.

iv $\frac{16}{8s + 4} = \frac{16/4}{8/4s + 1}$

$= \frac{4}{2s + 1}$ = 4 = D.C gain.
2 = Time Constant.

6. $\frac{\omega(s)}{\theta} = \frac{km}{\tau m s + 2}$

if $km = 15s^{-1}$
$\tau m = 4s$

$$\frac{15}{4s+2} = \frac{15/2}{4s/2+1} = \frac{7.5}{2s+1}$$

DC gain = $7.5s^{-1}$

Time Constant = 2.

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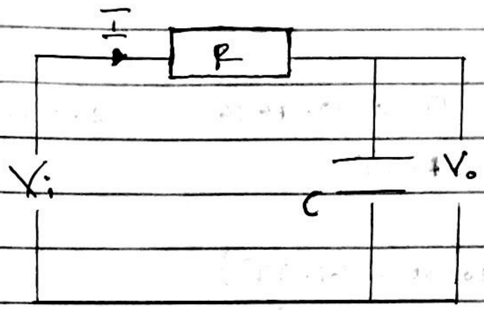
LINEAR SYSTEMS I

SYSTEM RESPONSE II.

1



$$\left(\frac{V_o}{V_i}\right)(s) = \frac{1}{Ts+1}$$ where $T = RC$

$R = 47\Omega$; $C = 20\mu F$

$V_i = 5\sin(2000t)$.

$$G(s) = \frac{1}{Ts+1}$$

$$G(j\omega) = \frac{1}{9.4\times10^{-4}j\omega+1} \times \frac{9.4\times10^{-4}j\omega-1}{9.4\times10^{-4}j\omega-1}$$

$$G(j\omega) = \frac{9.4\times10^{-4}j\omega-1}{(9.4\times10^{-4})^2\omega^2-1}$$

$$G(j\omega) = \frac{-1}{(9.4\times10^{-4})^2\omega^2-1} + \frac{9.4\times10^{-4}j\omega}{(9.4\times10^{-4})^2\omega^2-1}$$

$\omega = 2000\text{rad/s}$.

$$\phi = \tan^{-1}\left[\frac{\dfrac{9.4\times10^{-4}(2000)}{9.4\times10^{-4}\omega^2-1}}{\dfrac{-1}{9.4\times10^{-4}\omega^2-1}}\right] = \tan^{-1}\left[\frac{9.4\times10^{-4}(2000)}{-1}\right]$$

$$\theta = -61.99^\circ$$

$$|G(j\omega)| = \frac{1}{\sqrt{(9.4\times10^{-4})\omega^2 + 1^2}} = \frac{1}{\sqrt{(9.4\times10^{-4})^2(2000)^2 + 1^2}}$$

$$= 0.4696$$

V_o = amplitude = $5 \times 0.4696 = 2.348$

$\theta = 2000 - 67.99°$

$V_o = 2.348 \sin(2000t - 61.99°)$

$V_o = 2.35 \sin(2000t - 62°)$

2. $$\frac{x_o}{x_i} = \frac{1}{T^2s^2 + 2\delta Ts + 1}$$

$$G(s) = \frac{1}{(1 - T^2s^2) + 2\delta Ts}$$

$$G(j\omega) = \frac{1}{(1 - T^2\omega^2) + 2\delta Tj\omega}$$

$\delta = 0.2 \quad T = 0.4s \quad \omega = 2.5$ rad/s

$$G(j\omega) = \frac{1 - T^2\omega^2 - 2\delta Tj\omega}{(1 - T^2\omega^2)^2 + 4\delta^2T^2\omega^2}$$

$$= \frac{1 - (0.4)^2(2.5)^2 - 2(0.2)(0.4)(2.5)}{(1 - (0.4)^2(2.5)^2)^2 + 4(0.2)^2(0.4)^2(2.5)^2}$$

$G(j\omega) = 0 - 0.25$

$$\phi = \tan^{-1}\left[\frac{2.5}{0}\right] \simeq \infty$$

$\tan^{-1}(\infty) = \underline{90°}$

$$|G(j\omega)| = \sqrt{0^2 + (2.5)^2}$$

$$= 2.5$$

Amplitude $\Rightarrow 6 \times 2.5$

$$= \underline{15}$$