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19/ENG07/016.
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MATHS 102.

1. If $M = pi - 6j - 3K$ $N = 4i + 3j - K$ $O = i - 3j + 2K$, find the value of for which (a) M and N are perpendicular to each other
b. M, N and O are Coplanar.

Solution.

a. Perpendicular vectors = 0

$\therefore M \cdot N = 0$

$0 = (pi - 6j - 3k) \cdot (4i + 3j - k)$

$0 = 4p - 18 + 3$

$0 = 4p - 15$

$15 = 4p$

$p = \frac{15}{4}$ $\therefore p = 3\frac{3}{4}$ $\therefore p = 3\frac{3}{4}$

b. $M \cdot (N \times O) = 0$

$$M \cdot (N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$= p(6-3) + 6(8+1) - 3(-12-3)$

$= 3p + 54 + 45$

$= 3p + 99$

$0 = 3p + 99$

$-3p = 99$

$p = \frac{-99}{3}$

$p = -33$

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2. Find the direction cosines and the unit vector along the sum of $3i+2j+5k$, $2i-j+6k$, $5i+2j-3k$.

Solution.

i. Direction Cosines $= \cos\theta = \dfrac{\bar{A}\cdot\bar{B}\cdot\bar{C}}{|A|\,|B|\,|C|}$

$(\bar{A}\cdot\bar{B}\cdot\bar{C}) = (3i+2j+5k)\cdot(2i-j+6k)\cdot(5i+2j\cdot 3k)$

$= 30 - 4 - 90$

$(\bar{A}\cdot\bar{B}\cdot\bar{C}) = -64$

$|A| = \sqrt{(3)^2+(4)^2+(5)^2}$

$= \sqrt{9+16+25}$

$= \sqrt{50}$

$|A| = 5\sqrt{2}$

$|B| = \sqrt{(4)^2+(-1)^2+(6)^2}$

$= \sqrt{16+1+36}$

$|B| = \sqrt{53}$ $\therefore |B| = \sqrt{53}$

$|C| = \sqrt{(5)^2+(2)^2+(-3)^2}$

$= \sqrt{25+4+9}$

$|C| = \sqrt{50}$

$\cos\theta = \dfrac{-64}{\sqrt{50}\times\sqrt{53}\times\sqrt{50}} = \dfrac{-64}{364}$

$\cos\theta = -0.1758$

$\theta = \cos^{-1} -0.1758$

$\theta = 100^\circ$

ii. $(A+B+C) = (3i+2j+5k)+(2i-j+6k)+(5i+2j-3k)$

$= 10i+3j+8k$

$|(A+B+C)| = \sqrt{(10)^2+(3)^2+(8)^2}$

$= \sqrt{100+9+64} = \sqrt{173}$

$|(A+B+C)| = \sqrt{173}$

$\therefore$ Unit vector $(\hat{u}) = \dfrac{10i+3j+8k}{\sqrt{173}}$ $\therefore \hat{U} = \dfrac{10i}{\sqrt{173}} + \dfrac{3i}{\sqrt{173}} + \dfrac{8k}{\sqrt{173}}$

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3. If $F = 3ui + u^2j + (u+2)k$ and $V = 2ui - 3uj + (u-2)k$, evaluate the integral of $(F\times V)du$ from 0 to 1.

Solution.

$$F\times V = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$$= i\left[(u^3-2u^2)-(-3u^2-6u)\right] - j\left[(3u^2-6u)-(2u^2+4u)\right] + k\left[(-9u^2-2u^3)\right]$$

$$= \left[u^3+u^2+6u\right]i - \left[u^2-10u\right]j + \left[-9u^2-2u^3\right]k$$

$$\int (F\times V) = \left[\frac{u^4}{4}+\frac{u^3}{3}+\frac{6u^2}{2}\right]i + \left[-\frac{u^3}{3}+\frac{10u^2}{2}\right]j + \left(-\frac{9u^3}{3}-\frac{2u^4}{4}\right)k$$

$$= \left[\frac{u^4}{4}+\frac{u^3}{3}+3u^2\right]i + \left[-\frac{u^3}{3}+5u^2\right]j + \left[-3u^3-\frac{u^4}{2}\right]k$$

$$\int_0^1 (F\times V)du = \left[\frac{3+4+36}{12}\right]i + \left[\frac{-1+15}{3}\right]j + \left[\frac{-6-1}{2}\right]k + [0]^0$$

$$\int_0^1 (F\times V)du = \left(\frac{43}{12}\right)i + \left(\frac{14}{3}\right)j - \left(\frac{7}{2}\right)k$$

$$\int_0^1 (F\times V)du = 3.58i + 4.67j - 3.5k$$