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DEPT: ELECT/ELECT

EEE 324

ASSIGNMENT SOLN

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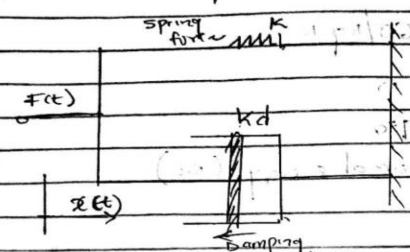
1)



Newton law of Spring force

$$= F(t) - K(x-0) - Kd\frac{d(x-0)}{dt} = 0$$

Converting to Laplace transform.

$$F(s) - Kx(s) - Kd\,s\,x(s) = 0$$

$$F(t) \longrightarrow \boxed{G(s) = \frac{x(s)}{F(s)} = \frac{1/K}{\tau s+1}} \longrightarrow x(t)$$

$$F(s) = Kx(s) + Kd\,s\,x(s) = [K + Kds]x(s)$$

$$G(s) = \frac{x(s)}{F(s)} = \frac{\cancel{x(s)}}{[K+Kds]\cancel{x(s)}}$$

$$G(s) = \frac{1}{[K+Kds]}$$ comparing to $\frac{K}{s+a}$

$$G(s) = \frac{1/K}{1+(Kd/K)s}$$

If we replace Kd/K with τ — time constant

$$\frac{Kd}{K} = \tau$$

Recall $Kd = 0.03$, $K = 4\times10^{3}$

$$= \frac{0.03}{4\times10^{-3}}$$

$\tau = 0(75\times10^{-5}$ seconds

7.5×10^{-6} secs $\approx 7.5\mu s$

2.

mass = 0.5 kg

Specific heat capacity = 346 J/kg K.

$\theta_1 = 20°C$

$\theta_2 = 120°C$

θ after 6 minutes is 119°C

Using change in thermal energy (ΔE)

$\Delta E = mc\Delta\theta$

$E_2 = mc(\theta_2 - \theta_1)$

$E_1 = mc(\theta - \theta_1)$

$$G(s) = \frac{E_1}{E_2} = \frac{\cancel{mc}(\theta - \theta_1)}{\cancel{mc}(\theta_2 - \theta_1)}$$

$$G(s) = \frac{\theta - \theta_1}{\theta_2 - \theta_1}$$

$$G(s) = \frac{E_1}{E_2} = \frac{\theta_1}{\theta_2} = \boxed{\frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{1}{Ts+1}}$$

$$\frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{1}{Ts+1}$$

cross multiply

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

Let us make $\theta_2 - \theta_1 = K(t)$

$$\theta - \theta_1 = \frac{K(t)}{Ts+1}$$

comparing to $\frac{K}{s+a}$

$$\theta - \theta_1 = \cancel{\frac{K(1/T)}{s+1/T}} \quad \frac{K(t)(1/T)}{s+1/T}$$

$K(t) = K/s$

$$\theta - \theta_1(t) = \frac{K(1/T)}{s(s+1/T)} \quad \text{Exponential growth}$$

$$\theta - \theta_1(t) = K\left[1 - e^{-t/T}\right]$$

Recall $K = \theta_2 - \theta_1$

$$(\theta - \theta_1) = (\theta_2 - \theta_1)\left[1 - e^{-t/T}\right]$$

making θ subject of formula

$$\theta = \theta_1 + (\theta_2 - \theta_1)\left[1 - e^{-t/T}\right]$$

$Ts+1$

to find T

Recall

$\theta_1 = 20^\circ$ $\theta_2 = 120$ $t = 6 \times 80 = 3600$ $\theta = 119$

$\therefore 119 = 20 + (120 - 20)\left[1 - e^{-\frac{3600}{T}}\right]$

$119 = 120 \left[1 - e^{3600/T}\right]$

$-1 = \left[1 - e^{3600/T}\right]$

$-2 = -e^{3600/T}$

$\ln(-2) = -3600$

$T + 2T = 5198.70$

3.

$$\frac{\omega}{K_m X} = \frac{1}{Ts + 1}$$

$$T = \frac{J}{K_3} \quad \text{and} \quad K_m = \frac{k_1 K_2}{K_3}$$

$$\frac{\omega}{K_m X} = \frac{1}{Ts + 1} \quad \langle \text{cross multiply} \rangle$$

$$\omega = \frac{K_m X}{Ts + 1}$$

The input (X) is a step size $X = 1/s$

$$\omega = \frac{K_m}{s}\left(\frac{1}{Ts + 1}\right)$$

$$\omega = \frac{K_m}{s}\left(\frac{1/T}{s + 1/T}\right)$$

$$\omega = K_m \left(\frac{1/T}{s(s + 1/T)}\right)$$

$$\frac{\frac{1}{T}}{s(s + 1/T)} \text{ — Exponential growth.}$$

$$\omega(t) = K_m X\left[1 - e^{-t/T}\right]$$

at $t = 0$ $K_m\left[1 - e^{0}\right] = 0 K_m X$

at $t = T$ $K_m\left[1 - e^{-T/T}\right] = 0.632 K_m$

at $t = 4T$ $K_m\left[1 - e^{-4T/T}\right] = 0.982 K_m$

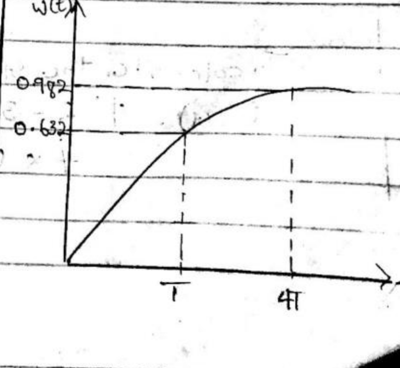
To calculate the % change

$\Delta\%$ at $T = [0.632 - 0] \times 100$

$= 63.2\%$

$\Delta\%$ at $4T = [0.982 - 0] \times 100$

$= 98.2\%$



4

$G(s) = \frac{1}{3s+1}$ $T = 3$

$C = 4mm/s$ $G(s) = \frac{\theta_o}{\theta_i}$ $\theta_i = \frac{C}{s^2}$

$\frac{\theta_o}{\theta_i} = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i}{3s+1}$

Substituting θ_i into $\theta_o(s)$

$\theta_o(s) = \frac{C}{s^2(3s+1)}$

$\theta_o(s) = \frac{C}{s^2} \cdot \frac{1}{(3s+1)}$

$\frac{1}{(3s+1)} = \frac{1/3}{s+1/3}$

$\theta_o(s) = \frac{C/3}{s^2(s+1/3)}$ → Exponentially graph

taking inverse laplace transform $\theta_o(t) = C[t - T(1-e^{-t/T})]$

$\theta_o(t) = C[t-3(1-e^{-t/T})]$

$e^{-t/T}$ is negligible

$\theta_o(t) = C[t-3(1)]$

$\theta_o(t) = Ct - 3C$

Steady state error $\theta_e = \theta_i - \theta_o$

$\theta_i(s) = \frac{C}{s^2}$, $\theta_i(t) = Ct$

$\theta_e = Ct - (Ct - 3C)$

$\theta_e = 3C$

To calculate error after 2 second

$\theta_e = 3 \times 4mm/s = 12mm$

To calculate the steady state error

$\theta_o(t) = 4[2-3(1-e^{-2/3})]$

4×0.54025

$= 2.161 mm$

i) $G(s) = \frac{2}{0.2s + 0.5}$ comparing to $\frac{K}{Ts+1}$

T - time constant

K - DC gain

to find DC gain and time constant

<Divide all through by 0.5>

$$\frac{2/0.5}{0.2s/0.5 + 0.5/0.5} = \frac{4}{0.4s+1}$$

DC gain is $4s^{-1}$ and 0.4 is Time constant

ii) $G(s) = \frac{0.2}{0.05s + 0.1}$

comparing to $\frac{K}{Ts+1}$

<Divide all through by 0.1>

$$\frac{\frac{0.2}{0.1}}{\frac{0.05s}{0.1} + \frac{0.1}{0.1}} = \frac{2}{0.5s+1} = \frac{K}{Ts+1}$$

DC gain is $2s^{-1}$ Time constant is 0.5 secs

iii) $G(s) = \frac{2}{3s+1}$ comparing to $K/Ts+1$

~~divide~~ $\frac{2}{3s+1} = \frac{K}{Ts+1}$

DC gain is $2s^{-1}$ Time constant is 3 secs

iv) $G(s) = \frac{16}{8s+4}$

comparing to $\frac{K}{Ts+1}$

<Divide all through 4>

$$\frac{16/4}{8s/4 + 4/4} = \frac{4}{2s+1} = \frac{K}{Ts+1}$$

DC gain is $4s^{-1}$ Time constant is 2secs

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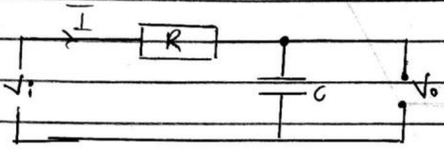
SYSTEM RESPONSE 2 ASSIGNMENT

$$\frac{V_o}{V_i}(s) = \frac{1}{Ts+1}$$

$T = RC$



$R = 47\Omega \quad C = 20\mu F = 20\times10^{-6}F$

$T = R\times C = 47\times20\times10^{-6} = 9.4\times10^{-4}$

$$G(s) \quad \frac{1}{Ts+1}$$

$s = j\omega$

$$G(j\omega) = \frac{1}{Tj\omega+1}$$

$$G(j\omega) = \frac{1}{9.4\times10^{-4}j\omega+1}$$

multiply by conjugate

$$\frac{1}{9.4\times10^{-4}j\omega+1} \times \frac{9.4\times10^{-4}j\omega-1}{9.4\times10^{-4}j\omega-1}$$

$$= \frac{9.4\times10^{-4}j\omega-1}{(9.4\times10^{-4}j)^2\omega^2-1}$$

$$G(j\omega) \quad \frac{j9.4\times10^{-4}\omega}{(9.4\times10^{-4})^2\omega^2-1} \qquad \frac{1}{(9.4\times10^{-4})^2\omega^2-1}$$

$$G(j\omega) = \frac{-1}{(8.836\times10^{-7})\omega^2-1} + j\,\frac{9.4\times10^{-4}\omega}{(8.836\times10^{-7})\omega^2-1}$$

$\omega = 2000\,rad/s$

$G(j\omega) = -0.39457 + j\,0.74179$

$\phi = \tan^{-1}(J/R)$

to find the phase angle

$$\phi = \tan^{-1}\left[\frac{9.4\times10^{-4}(2000)}{(8.836\times10^{-7})\omega^2 - 1}\right]$$

$$\phi = \tan^{-1}\left[\frac{9.4\times10^{-4}(2000)}{-1}\right]$$

$$= \tan^{-1}\left[\frac{1.88}{-1}\right]$$

$$\phi = -61.99^\circ$$

$$|G(j\omega)| = \frac{1}{\sqrt{(1.88)^2 + (-1)^2}}$$

$$|G(j\omega)| = 0.4696$$

Output amplitude is 5×0.4696

$= 2.3480$

$\simeq 2.35$

and the phase angle is -61.99°

comparing to $V_o \sin(\omega t + \phi)$

$V_o = 2.348 \sin(2000t - 61.99^\circ)$

$\simeq V_o\ 2.35 \sin(2000t - 62^\circ)\ V$

2

$$\frac{X_o}{X_i} = \frac{1}{T^2s^2 + 2\sigma Ts + 1}$$

T = 0.4 seconds $\sigma = 0.2$ $w = 2.5$ rad/s

$$G(j\omega) = \frac{1}{T^2(j\omega)^2 + 2T\sigma j\omega + 1}$$

$$G(j\omega) = \frac{1}{(1-T^2\omega^2) + 2T\sigma j\omega} \times \frac{(1-T^2\omega^2) - 2T\sigma j\omega}{(1-T^2\omega^2) - 2T\sigma j\omega}$$

multiplying by conjugate

$$G(j\omega) = \frac{(1-T^2\omega^2) - 2T\sigma j\omega}{(1-T^2\omega^2)^2 + 4T^2\sigma^2\omega^2}$$

$$= \frac{(1-(0.4)^2(2.5)^2) - j(2(0.4)(0.2)(2.5))}{(1-(0.4)^2(2.5)^2)^2 + 4(0.4)^2(0.2)^2(2.5)^2}$$

$$G(j\omega) = 0 - j2.5$$

$$\phi = \tan^{-1}\left(\frac{-2.5}{0}\right) = \tan^{-1}(\infty) = 90^\circ$$

$$|G(j\omega)| = \sqrt{0^2 + (-2.5)^2}$$

$$= 2.5$$

Phase shift = 90°

Amplitude = $\cancel{|G(j\omega)| \times A}$ from $\theta_i = 6\sin(\omega t)$

$= 2.5 \times 6$

$= 15$ v