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MECHATRONICS ENGINEERING
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Dr Oyelami / Mr Okunlola's class

1. $M = pi - 6j - 3k \qquad N = 4i + 3j - k \qquad O = i - 3j + 2k$

(a) $\bar{A} \cdot \bar{B} = 0$ — perpendicular vectors

$\therefore \bar{M} \cdot \bar{N} = 0$ ie $(pi - 6j - 3k) \cdot (4i + 3j - k) = 0$

$\Rightarrow 4p - 18 + 3 = 0$

$4p = 18 - 3$

$p = 15/4 = 3.75$

(b) $\bar{A} \cdot (\bar{B} \times \bar{C}) = 0$ — Coplanar vectors

$\therefore \bar{M} \cdot (\bar{N} \times \bar{O}) = 0$

$$\Rightarrow \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = p \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} - -6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + -3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$\Rightarrow p(6 - 3) - -6(8 - -1) - 3(-12 - 3) = 0$

$\Rightarrow p(3) + 6(9) - 3(-15) = 0$

$3p + 54 + 45 = 0 \qquad \Rightarrow 3p = 99$

$p = 99/3 = 33$

2 $\bar{A} = 3i + 2j + 5k \quad \bar{B} = 2i + j + 6k \quad \bar{C} = 5i + 2j - 3k$

$(\bar{A} + \bar{B} + \bar{C}) = 10i + 3j + 8k$

Direction cosine $= \dfrac{a_x}{|a|} = \cos\theta$

$|\bar{A} + \bar{B} + \bar{C}| = \sqrt{10^2 + 3^2 + 8^2} = \sqrt{173} = 13.15$

$a_x = 10 \quad a_y = 3 \quad a_z = 8$

$\cos\theta = \dfrac{10}{13.15} \Rightarrow \theta = \cos^{-1} 0.7603 = 40.51^\circ$

$\cos\theta = \dfrac{3}{13.15} \Rightarrow \theta = \cos^{-1} 0.2281 = 76.82^\circ$

$\cos\theta = \dfrac{8}{13.15} \Rightarrow \theta = \cos^{-1} 0.6082 = 52.54^\circ$

$\therefore$ ~~the~~ direction cosines are $a_x = 0.7603$

$a_y = 0.2281 \quad a_z = 0.6082$

Unit vector $= \dfrac{\bar{u}}{|u|} = \dfrac{10i + 3j + 8k}{13.15} = \cancel{\dfrac{10}{13.15} i + \dfrac{3}{13.15} j + \dfrac{8}{13.15} k}$

$= \dfrac{10}{13.15} i + \dfrac{3}{13.15} j + \dfrac{8}{13.15} k$

3 $F = 3ui + u^2 j + (u+2)k \quad V = 2ui - 3uj + (u-2)k$

$\int (F \times V)$

$(F \times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix} =$

$$i\left(u^2(u-2) - -3u(u+2)\right) - j\left(3u(u-2) - 2u(u+2\right.$$

$$+ k\left(-9u^2 - 2u^3\right)$$

$$= i\left(u^3 - 2u^2 + 3u^2 + 6u\right) - j\left(3u^2 - 6u - 2u^2 - 4u\right.$$

$$+ k\left(-9u^2 - 2u^3\right)$$

$$= i\left(u^3 + u^2 + 6u\right) - j\left(u^2 - 10u\right) + k\left(-9u^2 - 2\right.$$

$$\int (F \times V) = \int (u^3 + u^2 + 6u)\,i - \int (u^2 - 10u)\,j +$$

$$\int (-9u^2 - 2u^3)\,k$$

$$\int (F \times V) = \left(\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^2}{2}\right)i - \left(\frac{u^3}{3} - \frac{10\ldots}{2}\right.$$

$$+ \left(-\frac{9u^3}{3} - \frac{2u^4}{4}\right)k$$

$$\int (F \times V) = \left(\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right)i - \left(\frac{u^3}{3} - 5u^2\right)j$$

$$+ \left(-3u^3 - \tfrac{1}{2}u^4\right)k$$

$$\int_0^1 (F \times V) = \left[\left(\frac{1^4}{4} + \frac{1^3}{3} + 3(1)^2\right)i - \left(\frac{1^3}{3} - 5(1)^2\right)\right.$$

$$\left. + \left(-3(1)^3 - \tfrac{1}{2}(1)^4\right)\right] - \left[\left(\frac{0^4}{4} + \frac{0^3}{3} + 3\ldots\right.\right.$$

$$- \left(0^3/3 - 5(0)^2\right)j + \left(-3(0)^3 - \tfrac{1}{2}(0)^4\right)k\Big]$$

$$\int_0^1 (F \times V) = \left(\frac{1}{4} + \frac{1}{3} + 3\right)i - \left(\frac{1}{3} - 5\right)j + \left(-3 - \tfrac{1}{2}\right.$$

$$\int_0^1 (F \times V) = \frac{43}{12}i + \frac{14}{3}j - \frac{7}{2}k$$