System response 1

① from the diagram

$Spring = K(x-0)$

$f_{damper} = K_d \frac{d(x-0)}{dt}$

$f(t) \Rightarrow f(t)$

$f(t) - K(x-0) - K_d \frac{d(x-0)}{dt}$

$0 = f(t) - Kx - K_d \frac{dx}{dt}$

Taking the Laplace Transform

$\therefore f(s) = Kx(s) - K_d s\, x(s) = 0$

$f(s) = (K + K_d s)\, x(s)$

$G(s) = \frac{x(s)}{f(s)} = \frac{1}{K + K_d s}$

$\Rightarrow \frac{1/K}{1 + [K_d/K]s}$

$\frac{x}{f(s)} = \frac{1/K}{(K_d/K)s + 1} = \frac{1/K}{Ts+1}$

$T = K_d/K$

$= \frac{0.03}{4 \times 10^3}$ T

$T = 7.5 \times 10^{-6}$ seconds

X_0 after T seconds

$X_0 = f/K\,(1 - e^{-1})$

$= 100/4\times10^{-3}\,(1 - e^{-1})$

$= 100/4\times10^{3} \times 0.632$

$= 0.0158m$

$= 0.016m \simeq 16\times10^{-3} = 16mm$

Scanned with CamScanner

(2) $E_2 = mcD\theta \Rightarrow mc(\theta_2 - \theta_1)$

$E_1 = mc(\theta_2 - \theta_1)$

θ = Now Temperature

$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta_2-\theta_1)}{mc(\theta_2-\theta_1)} = \frac{1}{Ts+1}$$

$$\frac{\theta-\theta_1}{\theta_2-\theta_1} = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2-\theta_1}{Ts+1}$$

let $\theta_2 - \theta_1 (s) = k(t)$

$$(\theta - \theta_1)(s) = \frac{k(t)}{Ts+1}$$

then

$$(\theta - \theta_1 (s) = \frac{k(t)\,(1/T)}{s + 1/T}$$

Laplace transform, D.f $kd\,t$

$= k/s$

$$(\theta-\theta_1)(s) = \frac{k(1/T)}{s(s+1/T)}$$

$LT^{-1}(\theta-\theta_1) = k(1-e^{-t/T})$

$\theta(t) = \theta_1 + (\theta_2 - \theta_1)(1-e^{-t/T})$

$119 = 20 + (120-20)(1-e^{-t/T})$

$99 = 100(1-e^{-t/T})$

$0.99 = (1-e^{-t/T}$

$0.99 - 1 = -e^{-t/T}$

$\ln 0.01 = -t/T$

$-4.605 = -6/T$

$T = 6/4.605 = 1.302$ mins or 78.17 seconds

Themal capacitance $C = mC = 0.5 \times 34.6 = 173 \, J/K$

$T = RC$

$R = T/C$

$R = \frac{78.17}{173} = 0.451 \, K/N$

(3) a)

$$\frac{\omega}{K_m x} = \frac{1}{Ts+1}$$

$$T = 1/k_s \qquad k_m = \frac{k_1 k_2}{k_s}$$

$$\omega = \frac{k_m x}{Ts+1}$$

Laplace Transform of the step input

$$\omega = \frac{k_m x}{s}\left(\frac{1}{Ts+1}\right)$$

$$\frac{k_m x}{s}\left(\frac{1/T}{s+1/T}\right)$$

$$\omega(s) \Rightarrow k_m x\left(1-e^{-t/T}\right)$$

at $t=0$ $\quad k_m x\left(1-e^{0}\right) = $ Initial

at $t=T$ $\quad k_m x\left(1-e^{-t/T}\right) = 0.63\, k_m x$

at $t=4T = k_m x\left(1-e^{-4T/4}\right)$

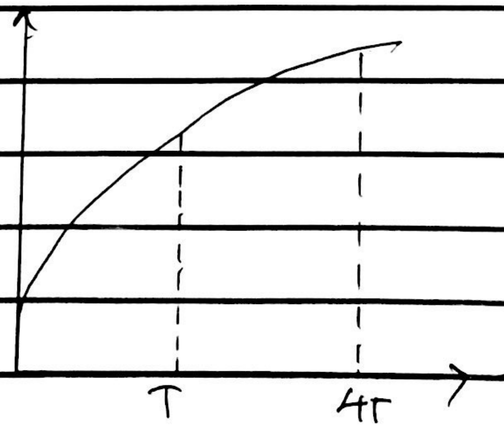
$\approx 0.981\, k_m x$

for $t=T$

$\Delta \% = (0.632-0)\times 100\% = 63.2\%$

$t = 4T$

$\Delta\% = (0.981-0)\times 100\% = 98.1\%$



(4) $\frac{\theta_o}{\theta_i}(s) = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$

$\theta_o(s) = \frac{c}{S^2(3s+1)}$

$\theta_o(t) = \frac{c/3}{S^2(s+1/3)}$

$\theta(t) = C(t - 3C(1 - e^{-t/3}))$

where $t \gg s$ lox

$\theta_o(t) = C(t - 3(1))$

$\theta_o(t) = Ct - 3C$

$\theta_E - \theta_2 - \theta_o = Ct - (Ct - 3C) = 3C$

$T = 3 \quad C = 4mm/s$

after 2 seconds

$\theta = 4 \times 2 = 8mm$

$\theta_2 = 4mm \times 3 = 12mm$ at steady state

$\theta_o = 4(2 - 3(1 - e^{2/3}))$

$= 2.161mm$

(5)(i) $\frac{2}{0.2s+0.5} = \frac{2/0.5}{\frac{0.2s}{0.5}+1}$

$\Rightarrow \frac{4}{0.4s+1}$

4 = Dc gain

0.4 = Time constant

(ii) $\frac{0.2}{0.05s+0.1} = \frac{0.2/0.1}{\frac{0.05s}{0.1}+1}$

$= \frac{2}{0.5s+1}$

2 = Dc gain

0.5 = time constant

(iii) $\frac{2}{3s+1}$

2 = Dc gain

3 = Time constant.

(iv) $\frac{16}{8s+4} = \frac{16/4}{8/4 s+1}$ 3

$= \frac{4}{2s+1}$

4 = Dc gain

2 = Time constant.

Scanned with CamScanner

(b) $\frac{\omega}{\theta}(s) = \frac{K_m}{T_m s + 2}$

$K_m = 15 s^{-1}$

$T_m = 4$

$= \frac{15}{4s+2} = \frac{15/2}{4s/2 + 2/2}$

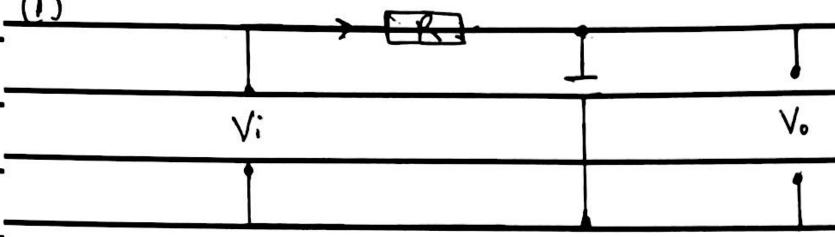
$= \frac{15/2}{4s/2 + 1} = \frac{7.5}{2s+1}$

D.c gain $= 7.5 \, ms^{-1}$

Time constant $= 2$ seconds

Systom response 2

(1)



$T = RC$

$R = 47\Omega \qquad C = 20\mu F$

$V_i = 5\sin(2000t)$

$T = 47 \times 20 \times 10^{-6} = 9.4 \times 10^{-9}$

$$\left[\frac{V_o}{V_1}\right](s) = \frac{1}{Ts+1}$$

$$G(s) = \frac{1}{Ts+1}$$

$$G(\omega) = \frac{1}{9.4\times10^{-9} j\omega + 1} \times \frac{9.4\times10^{-9} j\omega - 1}{9.4\times10^{-9} j\omega - 1}$$

$$G(\omega) = \frac{9.4\times10^{-9} j\omega - 1}{(9.4\times10^{-9})^2 \omega^2 - 1}$$

$$G(\omega) = \frac{-1}{(9.4\times10^{-9})\omega^2 - 1}$$

where $\omega = 2000$ rad/s

$$\phi = \tan^{-1}\left(\frac{9.4\times10^{-1}(2000)^2 - 1}{1}\right)$$

$$\phi = -61.99$$

$$|G(j\omega)| = \frac{1}{\sqrt{(9.4\times10^{-9})^2\omega^2 + 1^2}}$$

$$= \frac{1}{\sqrt{(9.4\times10^{-9})^2(2000)^2 + 1^2}}$$

$$= 0.4696$$

$V_o = 5 \times 0.4696 = 2.35$

(2) $\frac{x_o}{x_i} = \frac{1}{T^2s^2+2\delta Ts+1}$ G(s)

$$G(s) = \frac{1}{(1-T^2s^2)+2\delta Ts}$$

$$G(j\omega) = \frac{1}{(1-T^2\omega^2)+2\delta T j\omega}$$

$\delta = 0.2$ $T = 0.4s$ $\omega = 2.5 rad/s$

$$G(j\omega) = \frac{1-T^2\omega^2-2\delta Tj\omega}{(1-T^2\omega^2)+4\delta^2T^2\omega^2}$$

$$= \frac{1-(0.4)^2(2.5)^2-2(0.2)(0.4)(2.5)}{(1-(0.4)^2(2.5)^2)-4(0.2)^2(0.4)(2.5)^2}$$

$G(j\omega) = \theta = 2.5$

$$\phi = \tan^{-1}\left(\frac{2.5}{0}\right) = \infty$$

$\tan^{-1}(\infty) = 90°$

$$|G_{j\omega}| = \sqrt{0^2+(2.5)^2}$$

$= 2.5$

Amplitude $\Rightarrow 6 \times 2.5$

$= 15$