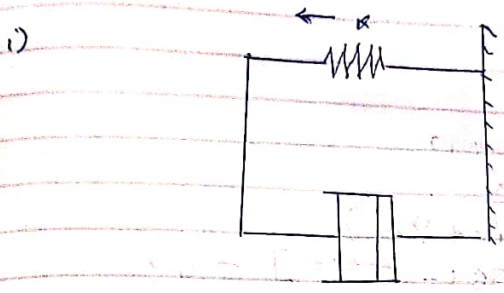


Assignment Qno



f_s Spring $\Rightarrow K(x-0)$
 f_{damp} $\Rightarrow kd \frac{d(x-0)}{dt}$
 $f_{ext} \Rightarrow f(t)$
 Newton's law $\Rightarrow f(t) - K(x-0) - kd \frac{d(x-0)}{dt} = 0$

$$0 = f(t) - Kx - kd \frac{dx}{dt} \quad \text{Laplace transform} \Rightarrow F(s) - Kx(s) - kd s x(s)$$

$$- kd s x(s)$$

$$f(s) = Kx(s) + kd s x(s)$$

$$F(s) = [K + kd s] X(s)$$

$$X(s) = \frac{F(s)}{K + kd s} = \frac{1}{K + kd s} \Rightarrow \frac{1/K}{1 + [kd/K]s}$$

Compare to $\frac{1}{Ts + 1}$

$$T = \frac{kd}{K} = \frac{0.03}{4 \times 10^3} = 0.75 \times 10^{-5} \text{ seconds}$$

$$T = 7.5 \times 10^{-6} \text{ s} = 7.5 \mu\text{s}$$

4)

$$\theta_0(t) = 0.6$$

$$\frac{d\theta_0(t)}{dt} = \frac{c}{s^2}$$

$$\frac{\theta_0(t)}{\theta_0} = \frac{1}{3s+1}$$

$$\theta_0(s) = \frac{\theta_0}{3s+1}$$

$$\theta_0(s) = \frac{c}{s^2(3s+1)}$$

$$\theta_0(s) = \frac{c/3}{s^2(3s+1)}$$

$$\theta_0(t) = c [t - 3e^{-t/3} - e^{-t/3}] - c'$$

where t is ω

$$\theta_0(t) = c [t - 3e^{-t/3}]$$

$$\theta_0(t) = ct - 3c$$

$$\theta_2 = \theta_1 - \theta_0 = ct - (ct - 3c) = 3c$$

$$T = 3 \quad c = 4 \text{ mm/s}$$

after 2 seconds

$$\theta_1 = 4 \times 2 = 8 \text{ mm}$$

$$\theta_2 = c \times 3 = 4 \text{ mm/s} \times 3 = 12 \text{ mm}$$

θ_0 from (1)

$$\theta_0 = 4 [2 - 3(1 - e^{-2/3})] = 2.161 \text{ mm}$$

$$5 \quad \frac{2}{0.25 + 0.5} = \frac{2/0.05}{0.025/0.5 + 1}$$

$$\Rightarrow \frac{4}{0.75 + 1} \quad \text{Compare to } \frac{K}{Tst + 1}$$

$4 = \text{D.C gain}$

$0.4 = \text{Time constant}$

$$ii) \frac{0.2}{0.05s + 0.1} = \frac{0.2/0.1}{0.05/0.1 + 0.1/0.1} = \frac{2}{0.5s + 1}$$

$$2 = D.C \text{ gain} \quad 0.5 = \text{Time Constant}$$

$$iii) \frac{2}{3s + 1} \Rightarrow 2 = D.C \text{ gain} \\ 3 = \text{Time Constant}$$

$$\frac{16}{8s + 4} \Rightarrow \frac{16/4}{8/4s + 1} = \frac{4}{2s + 1}$$

$$4 = D.C \text{ gain} \quad 2 = \text{Time Constant}$$

$$Q \quad \frac{C(s)}{D(s)} = \frac{K_m}{T_m s + 2}$$

$$K_m = 15s^{-1}$$

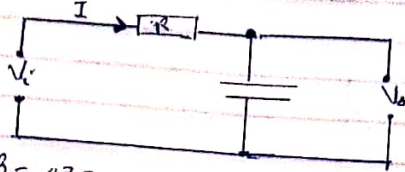
$$T_m = 4s$$

$$\Rightarrow \frac{15}{4s + 2} \Rightarrow \frac{15/2}{4s/2 + 1} = \frac{7.5}{2s + 1}$$

$$D.C \text{ gain} = 7.5s^{-1} \quad \text{Time Constant} = 2$$

Assignment Two

- 1) An electric circuit has a resistor and capacitor as shown. Show that the transfer function is $(V_o/V_i)(s) = 1/(Ts + 1)$ where $T = RC$



Given that $R = 47\Omega$ and $C = 20\mu F$ determine the output voltage when the output voltage when the input is sinusoidal such that $V_i = 5 \sin(2000t)$

Solution

$$\frac{V_o}{V_i} = \frac{1}{Ts + 1}$$

$$G(s) = \frac{1}{Ts + 1}$$

$$G(j\omega) = \frac{1}{9.9 \times 10^{-9} j\omega + 1} \times \frac{9.9 \times 10^{-9} j\omega - 1}{9.9 \times 10^{-9} j\omega - 1} = \frac{9.9 \times 10^{-9} j\omega - 1}{(9.9 \times 10^{-9} j\omega - 1)}$$

$$G(j\omega) = \frac{9.9 \times 10^{-9} j\omega - 1}{(9.9 \times 10^{-9} j\omega - 1)}$$

$$G(j\omega) = \frac{-1}{(9.9 \times 10^{-9})^2 \omega^2 - 1} + \frac{9.9 \times 10^{-9} j\omega}{(9.9 \times 10^{-9})^2 \omega^2 - 1}$$

$$\phi = \tan^{-1} \left[\frac{9.9 \times 10^{-9} (2000)}{9.9 \times 10^{-9} \omega^2 - 1} \right]$$

$$\phi = -61.99^\circ$$

$$|G(j\omega)| = \frac{1}{\sqrt{(9.4 \times 10^{-9})^2 \omega^2 + 1}} \rightarrow \frac{1}{\sqrt{(9.4 \times 10^{-9})^2 (2000)^2 + 1}} = 0.46$$

$$V_o = \text{amplitude} = 5 \times 0.4696 = 2.348$$

$$\phi = 2000t - 61.99^\circ$$

$$V_o = 2.348 \sin(2000t - 61.99^\circ)$$

$$V_o = 2.35 \sin(2000t - 62^\circ) \text{ V}$$

2) A
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A standard second order system has the transfer function $X_o/X_i = 1/(T^2s^2 + 2\zeta Ts + 1)$. The time constant T is 0.4 seconds and the damping ratio $\zeta = 0.2$. The input is varied harmonically as $\theta_1 = 6 \sin(\omega t)$.

Solution

$$\frac{X_o}{X_i} = \frac{1}{T^2s^2 + 2\zeta Ts + 1} \quad G(j\omega) = \frac{1}{(1 - T^2\omega^2) + j2\zeta T\omega}$$

$$G(j\omega) = \frac{1}{(1 - T^2\omega^2) + j2\zeta T\omega}$$

$$\zeta = 0.2 \quad T = 0.4 \text{ sec} \quad \omega = 2.5 \text{ rad/s}$$

$$G(j\omega) = \frac{1 - T^2\omega^2 - j2\zeta T\omega}{(1 - T^2\omega^2)^2 + 4\zeta^2 T^2\omega^2}$$

$$= \frac{1 - (0.4)^2(2.5)^2 - j2(0.2)(0.4)(2.5)}{((1 - (0.4)^2(2.5)^2)^2 + 4(0.2)^2(0.4)^2(2.5)^2)}$$

$$G(j\omega) = 0 - j2.5$$

$$\phi = \tan^{-1} \left[\frac{2.5}{0} \right] \Rightarrow \tan^{-1}(\infty) = 90^\circ$$

$$|G(j\omega)| = \sqrt{0^2 + (2.5)^2}$$

$$= 2.5$$

$$\text{Amplitude} = 6 \times 2.5$$

$$= 15$$