System Response 1

Q:

1) From the diagram

$Spring \Rightarrow K(x-0)$

$f_{damper} \Rightarrow K_d \frac{d(x-0)}{dt}$

$f(t) \Rightarrow f(t)$

~~Newton's~~ $f(t) - K(x-0) - K_d \frac{d(x-0)}{dt}$

$0 = f(t) - Kx - K_d \frac{dx}{dt}$

taking the Laplace transform

$= f(s) - Kx(s) - K_d s\, x(s) = 0$

$f(s) = [K + K_d s]\, x(s)$

$G(s) = \frac{x(s)}{f(s)} = \frac{1}{K + K_d s}$

$\Rightarrow \frac{1/K}{1 + [K_d/K]s}$

(4) $\frac{\theta_o(s)}{\theta_i} = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$

$\theta_o(s) = \frac{C}{s^2(3s+1)}$

$\theta_o(t) = \frac{C/3}{s^2[s+1/3]}$

$\theta(t) = C[t-3C(1-e^{-t/3})] - eqn$

When t is lose

$\theta_o(t) = C[t-3(1)]$

$\theta_o(t) = Ct - 3C$

~~$\theta_e = \theta_i - \theta_o$~~ $= Ct -$

$\theta_e = \theta_i - \theta_o = Ct - (Ct - 3C) = 3C$

$T = 3 \quad C = 4mm/s$

after 2 seconds

$\theta_i = 4 \times 2 = 8mm.$

$\theta_e = C \times 3 = 4mm \times 3 = 12mm$ at steady state

(iv) $\frac{16}{8s+4} \Rightarrow \frac{16/4}{8/4 s+1} = \frac{4}{2s+1}$

4 = Dc gain 2 = Time constant

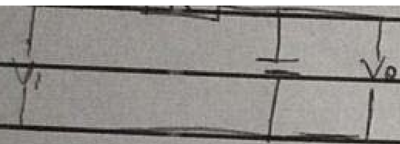
(b) $\frac{\theta}{V}(s) = \frac{Km}{Tm s+2}$

$Km = \cancel{1}\ 15s^{-1}$
$Tm = 4$

$= \frac{15}{4s+2} = \frac{15/2}{4s/2 + 2/2}$

$= \frac{15/2}{4s/2+1} = \frac{7.5}{2s+1}$

Dc gain = 7.5
Time constant = 2



$T = RC$

$R = 47\Omega \quad C = 20\mu F$

$V_i: 5\sin(2000t)$

$$\left[\frac{V_o}{V_i}\right](s) = \frac{1}{Ts+1}$$

$$T = 47 \times 20 \times 10^{-6}$$
$$= 9.4 \times 10^{-9}$$

$$G(s) = \frac{1}{Ts+1}$$

Recall $G(s) = G(j\omega)$

$$\cancel{G(j\omega) = \frac{1}{9.4\times10^{-9} j\omega +}}$$

$$G(j\omega) = \frac{1}{9.4\times10^{-9} j\omega + 1} \times \frac{9.4\times10^{-9} j\omega - 1}{9.4\times10^{-9} j\omega - 1}$$

$$\cancel{G(j\omega) = \frac{1}{9.4\times10^{-9}}}$$

$$G(j\omega) = \frac{9.4\times10^{-9} j\omega - 1}{(9.4\times10^{-9})^2\omega^2 - 1}$$

$$G(j\omega) = \frac{-1}{(9.4\times10^{-9})^2\omega^2 - 1} + \frac{9.4\times10^{-9} j\omega}{(9.4\times10^{-9})^2\omega^2 - 1}$$

where $\omega = 2000$ rad/s

(2) $\frac{x_o}{x_i} = \frac{1}{T^2s^2 + 2\delta Ts + 1} = G(s) = \frac{1}{[1 - T^2s^2] + 2\delta Ts}$

$G(j\omega) = \frac{1}{(1 - T^2\omega^2) + 2\delta Tj\omega}$

where $\delta = 0.2$ $T = 0.4s$ $\omega = 25 rad/s$.

$G(j\omega) = \frac{1 - T^2\omega^2 - 2\delta Tj\omega}{(1 - T^2\omega^2) + 4\delta^2T^2\omega^2}$

$= \frac{1 - (0.4)^2(2.5)^2 - 2(0.2)(0.4)(2.5)}{(1 - (0.4)^2(2.5)^2) - 4(0.2)^2(0.4)(2.5)^2}$

2.5

$G(j\omega) = 0 - 2.5$

$\phi = \tan^{-1}\left[\frac{2.5}{0}\right] = \tan^{-1}(\infty) = \underline{90^\circ}$

$|G(j\omega)| = \sqrt{0^2 + (2.5)^2}$

$= 2.5$

amplitude => 6×2.5

$= \frac{15}{2}$

$\cancel{\phi = \tan^{-1}[}$

Having the Same denominator it will Cancel out

$$\phi = \tan^{-1}\left[\frac{9.4\times10^{-1}(2000)^2 - 1}{1}\right]$$

$\phi = -61.99$

$$|G(j\omega)| = \frac{1}{\sqrt{(9.4\times10^{-9})^2\omega^2 + 1^2}}$$

$$= \frac{1}{\sqrt{(9.4\times10^{-9})^2(2000)^2 + 1^2}} = 0.4696.$$

$\cancel{V_o = 9m}$

$V_o = 5\times0.4696$

No 2

$E_2 = Mc\Delta\theta \Rightarrow mc(\theta - \theta_1)$

$= E_1 = mc(\theta_2 - \theta_1)$

θ = New temperature

$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta - \theta_1)}{mc(\theta_2 - \theta_1)} = \frac{1}{Ts+1}$$

$$\frac{\theta - \theta_1 (s)}{\theta_2 - \theta_1} = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

let $\theta_2 - \theta_1 (s) = K(t)$ ... (1)

$$(\theta - \theta_1)(s) = \frac{K(t)}{Ts+1}$$

then

$$(\theta - \theta)(s) = \frac{K(t)[1/T]}{s + 1/T}$$

Laplace transform of $K(t)$

$= K/s$

$$(\theta - \theta)(s) = \frac{K(1/T)}{s(s + 1/T)}$$

$L^{-1}[\theta - \theta_1] = K[1 - e^{-t/T}]$

$\theta_o = 4[2-3[1-e^{-2/3}]] = 2.161mm$

$\theta_e = \theta_i - \theta_o = 8mm - 2.161mm$

$= \cancel{5.1}\ \ 5.839mm$

(5) $\dfrac{2}{0.2s+0.5} \Rightarrow \dfrac{2/0.5}{0.2s/0.5+1}$

$\Rightarrow \dfrac{4}{0.4s+1}$ compare with the equation $\dfrac{K}{\tau s+1}$

4 = Dc gain 0.4 = Time Constant

(ii) $\dfrac{0.2}{0.05s+0.1} = \dfrac{0.2/0.1}{0.05s/0.1+1}$

$= \cancel{0.2 \div 0.1}\ \ \cancel{0.2/0.1}$

$\dfrac{2}{0.5s+1}$

2 = Dc gain 0.5 = time constant

(iii) $\dfrac{2}{3s+1} \Rightarrow$ 2 = Dc gain

3 = Time constant

(2)

(3) $\frac{\omega}{K_m x} = \frac{1}{Ts+1}$

$T = 1/k_3$ $\quad K_m = \frac{k_1 k_2}{k_3}$

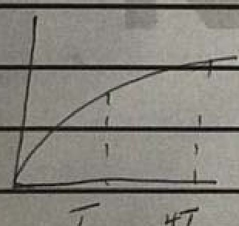
$\omega = \frac{K_m x}{Ts+1}$

Laplace transform of the step Input

$\omega = \frac{K_m x}{s}\left[\frac{1}{Ts+1}\right]$

$\frac{K_m x}{s}\left[\frac{1/T}{s+1/T}\right]$

$\omega(s) \Rightarrow K_m x\left[1-e^{-t/T}\right]$



at $t=0$ $\quad K_m x\left[1-e^{0}\right] =$ Initial

at $t=T$ $\quad K_m x\left[1-e^{-1T/T}\right] = 0.63 K_m x$

at $t = 4T = K_m x\left[1-e^{-4T/x}\right] =$

$0.981 K_m x$

For $t=T$

$\Delta\% = [0.632-0]\times 100\% = 63.2\%$

$t = 4T$

$\Delta\% = [0.981-0]\times 100\% = 98.1\%$

#

#

System Response 1

No1 Completion

$$F = Kx + Kd\frac{dx}{dt}$$

$$F(s) = Kx + Kd\,s\,x = x(K + Kd\,s)$$

$$\frac{X}{F}(s) = \frac{1}{Kd\,s + K} = \frac{1/K}{(Kd/\cancel{K})s + 1}$$

$$= \frac{1/K}{Ts + 1}$$

$T = 0.03/4000 = 7.5 \times 10^{-6}$ Seconds $= 7.5\mu s$

$$X_o = (F/K)\{1 - e^{-t/T}\} =$$

$$(100/4000)(1 - e^{-1}) = 0.015m$$

#

(2b) $119 = 20 + (120 - 20)(1 - e^{-6/T}) = 20 + 100(1 - e^{-6/T})$

$99 = 100(1 - e^{-6/T})$

$0.99 = (1 - e^{-6/T})$

$0.01 = e^{-6/T}$

$-4.605 = -6/T$

$T = -6/-4.605$

$T = 1.302$ minutes

or

$T = 78.17s$

Thermal Capacitance $C = mc = 0.5 \times 346 = 173\ J/K$.

$T = RC$

$R = T/C$

$= \frac{78.17}{173} = 0.452\ K/W$

#