MADUAGWUNA LOTANNA

18/ENG04/081

LINEAR SYSTEM

LUTAWNA MADUAGWUNA

[illegible]/ENG/[illegible]

ELECT/ELECT

$$Spring = k(x - 0)$$

$$f_{damper} = k_d \frac{d(x-0)}{dt}$$

$$f(t) = F(t)$$

from newton's law $\rightarrow f(t) - k(x-0) - k_d \frac{(x-0)}{dt} = 0$

Taking laplace transform

$$F(s) = kx(s) - k_d s\, x(s) = 0$$

$$F(s) = [k + k_d s]\, x(s)$$

$$\frac{x(s)}{F(s)} = G(s) = \frac{1/k}{1 + [k_d/k]s} = \frac{1/k}{Ts + 1}$$

$$T = \frac{k_d}{k} = \frac{0.03}{4 \times 10^3} = 7.5\mu s$$

2) E_2 = new energy

E_1 = initial energy

$E_2 = mc\Delta\theta \quad = E_2 = mc(\theta - \theta_1)$

$E_1 = mc\Delta\theta \quad = E_2 = mc(\theta_2 - \theta_1)$

where θ is the new temp of the metal

$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta - \theta_1)}{mc(\theta_2 - \theta_1)} = \frac{1}{Ts+1}$$

$$\Rightarrow \frac{\theta - \theta_1}{\theta_2 - \theta_1}(s) = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

Let $\theta_2 - \theta_1(s) = k(t) \quad -(1)$

$$(\theta - \theta_1)(s) = \frac{k(t)}{Ts+1}$$

$$(\theta - \theta_2)(s) = \frac{k(1/T)}{s + 1/T}$$

taking the laplace transform of $k(t)$

$$(\theta - \theta_1)(s) = \frac{k(1/T)}{s(s + 1/T)}$$

inverse laplace transform

$$(\theta - \theta_1)(s) = k(1 - e^{-t/T})$$

from (1) $k = \theta_2 - \theta_1$

$$(\theta - \theta_1) = (\theta_2 - \theta_1)\{1 - e^{-t/T}\}$$

3) a) $\dfrac{\quad}{k_m u} = \dfrac{1}{Ts+1}$

$T = \dfrac{1}{k_3}$ $\quad k_m = \dfrac{k_1 k_2}{k_3}$

$\omega = \dfrac{k_m u}{Ts+1}$

Laplace transform of the step input

$\omega = \dfrac{k_m u}{s}\left(\dfrac{1}{Ts+1}\right)$

$\dfrac{k_m u}{s}\left(\dfrac{1/T}{s+1/T}\right)$

$\omega(t) = k_m u\,(1-e^{-t/T})$

at $t=0$ $\quad k_m u\,(1-e^{0}) =$ Initial

at $t=T$ $\quad k_m u\,(1-e^{-t/T}) = 0.63\,k_m u$

at $t=4T = k_m u\,(1-e^{-4t/T})$

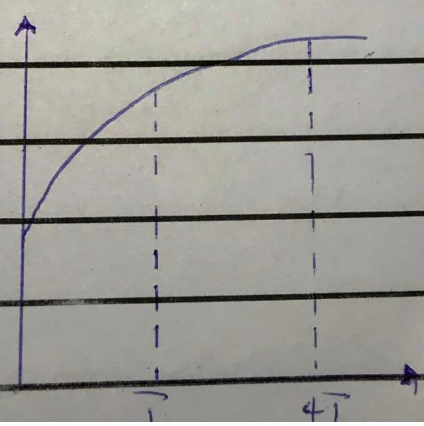
$= 0.981\,k_m u$

for $t = T$

$\Delta\% = (0.632-0) \times 100\% = 63.2\%$

$t = 4T$

$\Delta\% = (0.981-0) \times 100\% = 98.1\%$



5) i) $\frac{2}{0.2 + 0.5} = \frac{2/0.5}{0.2s/0.5 + 1}$

$\frac{4}{0.4s + 1}$

4 = Dc gain

0.4 = Time constant

ii) $\frac{0.2}{0.05s + 0.1} = \frac{0.2/0.1}{0.05s/0.1 + 1}$

$= \frac{2}{0.5s + 1}$

2 = Dc gain

0.5 = time constant

iii) $\frac{2}{3s + 1}$

2 = Dc gain

3 = Time constant

iv) $\frac{16}{8s + 4} = \frac{16/4}{8/4 s + 1}$

$= \frac{4}{2s + 1}$

4 = Dc gain

2 = Time constant

b) $\frac{\omega}{\theta}(s) = \frac{K_m}{T_m s + 2}$

$K_m = 15 s^{-1}$

$T_m = 4$

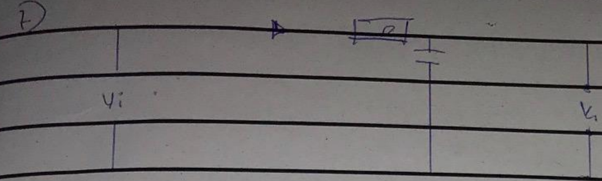
$= \frac{15}{4s+2} = \frac{15/2}{4s/2 + 2/2} = \frac{7.5}{2s+1}$

Dc gain $= 7.5 ms^{-1}$

Time constant $= 2$ seconds

System response 2 [illegible] write

7)



$T = RC$

$R = 47\Omega$ $\quad C = 20\mu F$

$V_i = 5\sin(2000t)$

$T = 47 \times 20 \times 10^{-6} = 9.4 \times 10^{-9}$

$$\left[\frac{V_o}{V_i}\right](s) = \frac{1}{Ts+1}$$

$$G(s) = \frac{1}{Ts+1}$$

$$G(j\omega) = \frac{1}{9.4\times10^{-9}j\omega+1} \times \frac{9.4\times10^{-9}j\omega-1}{9.4\times10^{-9}j\omega-1}$$

$$G(j\omega) = \frac{9.4\times10^{-9}j\omega-1}{(9.4\times10^{-9})^2\omega^2-1}$$

$$G(\omega) = \frac{-1}{(9.4\times10^{-9})\omega^2-1}$$

where $\omega = 2000$ rad/s $\quad \phi = \tan^{-1}\left(\frac{9.4\times10^{-1}(2000)^2-1}{1}\right)$

$\phi = -61.99$

$$G(j\omega) = \frac{1}{\sqrt{(9.4\times10^{-9})^2\omega^2+1^2}}$$

$$= \frac{1}{\sqrt{(9.4\times10^{-9})(2000)^2+1^2}}$$

$= 0.4696$

$V_o = 5 \times 0.4696 = 2.35$

$$\frac{x_o}{x_i} = \frac{1}{T^2s^2 + 2\delta Ts + 1}$$

$$G(s) = \frac{1}{(1 - T^2s^2) + 2\delta Ts}$$

$$G(j\omega) = \frac{1}{(1 - T^2\omega^2) + 2\delta Tj\omega}$$

$\delta = 0.2 \quad T = 0.4s \quad \omega = 25 \text{ rad/s}$

$$G(j\omega) = \frac{1 - T^2\omega^2 - 2\delta Tj\omega}{(1 - T^2\omega^2) + 4\delta^2T^2\omega^2}$$

$$= \frac{1 - (0.4)^2(2.5)^2 - 2(0.2)(0.4)(2.5)}{(1 - (0.4)^2(2.5)^2) - 4(0.2)^2(0.4)(2.5)^2}$$

$G(j\omega) = 0 = 2.5$

$$\theta = \tan^{-1}\left(\frac{2.5}{0}\right) = t$$

$\tan^{-1}(\infty) = 90°$

$$|G(j\omega)| = \sqrt{0^2 + (2.5)^2}$$

$= 2.5$

amplitude $\Rightarrow 6 \times 2.5$

$= 15$

f) $\frac{\theta_o(s)}{\theta_i} = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$

$\theta(s) = \frac{c}{s^2(3s+1)}$

$\theta_o(t) = \frac{c/3}{s^2(s+1/3)}$

$\theta(t) = c(t - 3c(1 - e^{-t/3}))$

where $t \neq 5$ log

$\theta_o(t) = c(t - 3(-))$

$\theta_o(t) = ct - 3c$

$\theta_t - \theta_2 - \theta_o = ct - (ct - 3c) = 3c$

$T = 3 \quad c = 4$ mm/s

after 2 seconds

$\theta = 4 \times 2 = 8$ mm

$\theta_2 = 4\text{mm} \times 3 = 12$ mm at steady state

$\theta_o = 4(2 - 3(1 - e^{2/3}))$

$= 2.161$ mm