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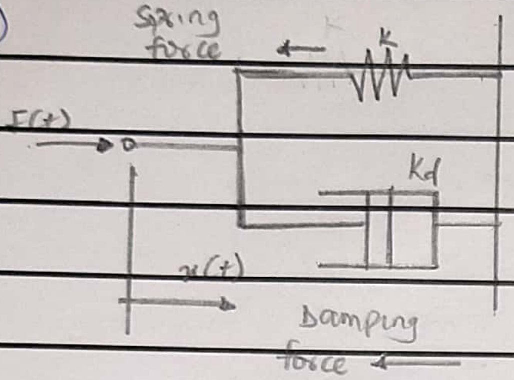
17ENG04/043

ELECTRICAL ELECTRONICS ENGINEERING

EEE 324 ASSIGNMENT. LINEAR SYSTEMS.

SYSTEM RESPONSE I.

1)



Given $G(s) = \dfrac{x(s)}{F(s)} = \dfrac{1/k}{Ts+1}$

$F_{spring} \Rightarrow k(x-0)$

$F_{damp} \Rightarrow k_d \dfrac{d(x-0)}{dt}$

Newtons law $= F(t) - K(x-0) - K_d \dfrac{d(x-0)}{dt} = 0$

$0 = F(t) - Kx - K_d \dfrac{dx}{dt}$

Getting the laplace transform

$F(s) - Kx(s) - K_d s\, x(s) = 0$

$F(s) = [K + K_d s]\, x(s)$

$G(s) = \dfrac{x(s)}{F(s)} = \dfrac{1}{K + K_d s} \rightarrow \dfrac{1/k}{1 + (k_d/k)s}$

Comparing to the Given $\dfrac{1/k}{Ts+1}$

$\tau = \dfrac{K_d}{K} = \dfrac{0.03}{4 \times 10^3} = 0.75 \times 10^{-5}$ seconds $= \underline{\underline{7.5\ \mu s}}$

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If a force $F = 100N$ calculate the value of x after T seconds

X_0 after T seconds

from $\frac{x}{F(s)} = \frac{1/K}{Ts + 1}$

$X_0 = F/K \, (1 - e^{-1})$

$= 100/4 \times 10^3 \, (1 - e^{-1})$

$= 0.025 \times 0.6321$

$= 0.0158 m$

$= 0.016 m = 16mm.$

$16 \times 10^{-3} = 16mm.$

2) Energy $= mc\Delta\theta$ 　　　 6 min = 360 sec

$E_2 = mc(\theta - \theta_1)$

$E_1 = mc(\theta_2 - \theta_1)$

$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta - \theta_1)}{mc(\theta_2 - \theta_1)} = \frac{1}{Ts+1}$$

$$G(s) = \frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

let $\theta_2 - \theta_1 = K(t)$

$$\theta - \theta_1 = \frac{K(t)}{Ts+1}$$

$$\theta - \theta_1 = \frac{K(1/T)}{s + 1/T}$$

Taking the laplace transform of $K(t)$

$= K/s$

$$\theta - \theta_1 = \frac{K(1/T)}{s(s + 1/T)}$$

inverse laplace

$\theta - \theta_1 = K[1 - e^{-t/T}]$

from above $K = \theta_2 - \theta_1$

$\theta - \theta_1 = \theta_2 - \theta_1 [1 - e^{-t/T}]$

$\theta = \theta_1 + (\theta_2 - \theta_1)[1 - e^{-t/T}]$ 　　 QED.

$119 = 20 + (120 - 20)[1 - e^{-360/T}]$

$119 - 20 = 100[1 - e^{-360/T}]$

$99 = 100 - 100e^{-360/T}$

$99 - 100 = -100e^{-360/T}$

Scanned with CamScanner

$$\frac{-1}{-100} = e^{-360/T}$$

$$0.01 = e^{-360/T}$$

$$\ln 0.01 = \frac{-360}{T}$$

$$T = \frac{-360}{\ln 0.01} = \underline{78.17\,sec}$$

Thermal resistance = T/C

$$C = mc = 0.5 \times 346 = 173\ J/K$$

$$R = \frac{78.17}{173} = \underline{0.452\ K/W}$$

3) $\dfrac{\omega}{K_m x} = \dfrac{1}{Ts+1}$

$T = \dfrac{J}{K_3}$ $\qquad K_m = \dfrac{K_1 K_2}{K_3}$

Given K_1, $K_2 x$, ω, J, K_3 and $K_1 p$

Assuming no load torque

Torque $= K_1 p$

$= Ja + K_3 \omega$

$K_1 K_2 X(s) = Js(\omega) + K_3 \omega$

$(K_1 K_2 / K_3) X(s) = \omega\{(J/K_3)s + 1\}$

$\dfrac{\omega}{K_m x} = \dfrac{1}{Ts+1}$

$K_1 K_2 x = J d\omega / dt + K_3 \omega$

$K_1 K_2 X(s) = \omega(Js + K_3)$

$K_m x(s) = \omega\{Ts+1\}$

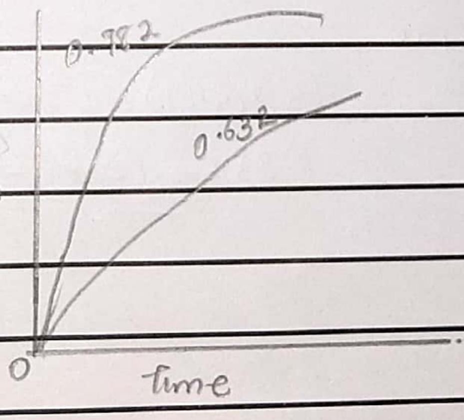
$\dfrac{\omega}{K_m} = \dfrac{x}{Ts+1}$ $\qquad$ Substituting $x(s) = 1/s$

$\dfrac{\omega}{K_m} = \dfrac{1}{s(Ts+1)}$

rearranging

$= \dfrac{1/T}{s(s+1/T)}$

$\dfrac{\omega}{K_m}(t) = 1 - e^{-t/T})$



when $t = T$ $w(t) =$

$K_m(1 - e^{-1}) = 0.632 K_m$

$= 0.632 \times 100 = 63.2\%$

when $t = 4T$ $w(t) =$

$K_m(1 - e^{-4}) = 0.982 K_m$

$= 0.982 \times 100 = 98.2\%$

$\theta_i(t) = ct \quad \theta_i(s) = c/s^2$

(4) $G(s) = \frac{1}{3s+1} = \frac{\theta_o}{\theta_i} = \frac{1}{Ts+1}$

$$\frac{\theta_o}{\theta_i} = \frac{1}{3s+1}$$

$$\theta_o(s) \quad \frac{\theta_i(s)}{3s+1} = \frac{c}{s^2(3s+1)}$$

$$\theta_o(s) = \frac{c/3}{s^2(3s/3 + 1/3)} = \frac{c/3}{s^2(s+1/3)}$$

$$\theta_o(t) = c[t - 3(1 - e^{-t/3})]$$

given $c = 4$ degree $t = 2$ seconds $T = 3$

at 2 seconds $\theta_i = 2 \times 4 = 8$ degree

$$\theta_o(t) = 4(2 - 3(1 - e^{-2/3})) = 8 - 5.839$$

$$\theta_o(t) = 2.161 \text{ degrees} = \underline{2.16 mm}$$

$$\theta_e = 8 - 2.161$$

$$= 5.839 \text{ degrees} = \underline{5.839 mm}$$

The steady state error is cT

where $T = 3 \quad c = 4$

$$\theta_e = 4 \times 3 = 12 \text{ degree}.$$

$$= \underline{12 mm}.$$

5) (i) $G(s) = \dfrac{2}{0.2s + 0.5} = \dfrac{2/0.5}{\frac{0.2s}{0.5} + 0.5/0.5}$

$= \dfrac{4}{0.4s + 1}$ comparing to $\dfrac{K}{Ts+1}$

D.C gain is $4\ s^{-1}$

Time constant is 0.4 seconds

(ii) $G(s) = \dfrac{0.2}{0.05s + 0.1} = \dfrac{0.2/0.1}{\frac{0.05}{0.1}s + 0.1/0.1}$

$= \dfrac{2}{0.5s + 1}$ comparing to $\dfrac{K}{Ts+1}$

DC gain is $2\ s^{-1}$

Time constant is 0.5 seconds

(iii) $G(s) = \dfrac{2}{3s+1} = 2/1$

Comparing to $\dfrac{K}{Ts+1}$

DC gain is $2\ s^{-1}$

Time constant is 3 seconds

(iv) $G(s) = \dfrac{16}{8s+4} = \dfrac{16/4}{\frac{8}{4}s + 4/4}$

$= \dfrac{4}{2s+1}$ comparing to $\dfrac{K}{Ts+1}$

DC gain is $4\ s^{-1}$

Time constant is 2seconds

6) $\frac{\omega(s)}{\theta} = \frac{K_m}{T_m s + 2}$ $K_m = 15s^{-1}$ $T_m = 4s$

$\frac{15}{4s+2} = \frac{15/2}{4/2 s + 2/2}$

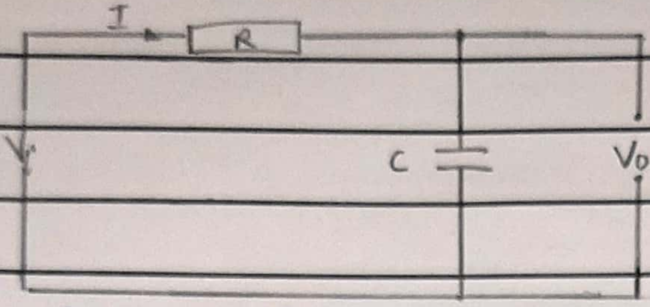
$= \frac{7.5}{2s+1}$ Comparing to $\frac{K}{T_s + 1}$

DC gain is $7.5s^{-1}$

Time constant is 2seconds

SYSTEM RESPONSE II

(1)



Given $R = 47\Omega$ $\quad C = 20\mu F$ $\quad V_i = 5\sin(2000t)$

$G(s) = \frac{V_o(s)}{V_i} = \frac{1}{Ts+1}$ where $T = RC$

$T = 47 \times 20 \times 10^{-6}$

$= 9.4 \times 10^{-4} s$

$V_i = IR + I/Cs$

$V_i = I(R + 1/C_s)$

$V_o = I/C_s$

$G(s) = \frac{V_o}{V_i}(s) = \frac{1/C_s}{I(R + 1/C_s)} = \frac{1}{RC_s+1}$

$\phi = -\tan^{-1}(\omega T) = -\tan^{-1}(2000 \times 9.40 \times 10^{-4}) = -42°$

$\frac{\theta_o}{\theta_i} = \frac{1}{\sqrt{1+T^2\omega^2}} = \frac{1}{\sqrt{1+(9.4\times10^{-4})^2 2000^2}} = 0.47$

Therefore

$\theta_o = (5 \times 0.47)\sin(2000t - 62°)$

$= 2.35\sin(2000t - 62°)$

2) $X_o/X_i = 1/(T^2s^2 + 2\delta Ts + 1)$

$T = 0.4$ seconds

$\delta = 0.2$

$\theta_1 = 6\sin(\omega t)$ at 2.5 rad/s

looking for amplitude and phase shift.

$$C = \frac{(1-\omega^2T^2)}{\{(1-\omega^2T^2) + (2\delta\omega T)^2\}}$$

$$C = \frac{1 - 2.5^2\,0.4^2}{\{1-(2.5^2\,0.4^2) + (2\times0.2\times2.5\times0.4)^2\}}$$

$$C = \frac{0}{\{0 + (0.4)^2\}} = 0$$

$$D = \frac{2\delta T\omega}{\{(1-T^2\omega^2)^2 + (2\delta\omega T^2)^2\}}$$

$$= \frac{2\times0.2\times0.4\times2.5}{\{(1-0.4^2\,2.5^2)^2 + (2\times0.2\times2.5\times0.4)^2\}}$$

$$= \frac{0.4}{0 + 0.4^2}$$

$$= \frac{0.4}{0.16}$$

$D = 2.5$

$\phi = -\tan^{-1}(D/C) = -\tan^{-1}(\infty)$ therefore $\theta = 90^\circ$

$|\theta_o|\theta_1| = \sqrt{D^2 + C^2}$

$= \sqrt{2.5^2 + 0^2}$

$= 2.5$

$\theta = 6\times2.5$

$= 15^\circ$