BILIAMEEN ADEDOLAPO ABDULFATTAH 19/ENG05/019 MECHATRONICS MR OKUNLOLA

1 $M = pi - 6j - 3k$, $N = 4i + 3j - k$, $O = i - 3j + 2k$,

Vector M and N are perpendicular if

$$\bar{M} \cdot \bar{N} = 0$$

$\therefore$ $(pi - 6j - 3k) \cdot (4i + 3j - k) = 0$

$$4p - 18 + 3 = 0$$

$$4p - 15 = 0$$

$$4p = 15$$

$$p = \cancel{15/4}$$

$$p = 3\tfrac{3}{4} \quad 3\tfrac{3}{4}$$

ii Vector $\bar{M}, \bar{N}$ and $\bar{O}$ are coplanar if

$$\bar{M} \cdot (\bar{N} \times \bar{O})$$

$$\bar{M} \cdot (\bar{N} \times \bar{O}) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} - (-6)\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$$p(6 - 3) + 6[8 - (-1)] - 3(-12 - 3)$$

$$p(3) + 6(9) - 3(-15)$$

$$3p + 54 + 45 = 0$$

$$\bar{M} \cdot (\bar{N} \times \bar{O}) = 3p + 54 + 45 = 0$$

$$3p + 99 = 0$$

$$p = \cancel{-99/3} \quad p = -33$$

2 $\bar{A} = 3i + 2j + 5k$, $\bar{B} = 2i - j + 6k$ and $C = 5i + 2j - 3k$

Solution

$\bar{R} = \bar{A} + \bar{B} + \bar{C}$

$\bar{R} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$\bar{R} = (10i + 3j + 8k)$

Unit vector $e_R = \dfrac{\bar{R}}{|R|}$

$|R| = \sqrt{10^2 + 3^2 + 8^2} = \sqrt{173}$ #

$e_R = \dfrac{10i + 3j + 8k}{\sqrt{173}}$

b Direction cosine

$\cos\alpha = \dfrac{a_x}{|R|}$

$\cos\beta = \dfrac{a_y}{|R|}$

$\cos\gamma = \dfrac{a_z}{|R|}$

$|R| = \sqrt{173}$

$a_x = 10 \quad a_y = 3 \quad a_z = 8$

$\cos\alpha = \dfrac{10}{\sqrt{173}} = 0.7603$

$\cos\beta = \dfrac{3}{\sqrt{173}} = 0.2280$

Scanned by TapScanner

$$\cos \gamma = \frac{8}{\sqrt{173}} = 0.6082.$$

3 $F = 3ui + u^2 j + (u+2)k \quad V = 2ui - 3uj + (u-2)k$

$$\int (f \times v)\, du$$

Solution

$$\bar{f} \times \bar{v} = \begin{vmatrix} i & j & k \\ 3u & u^2 & u+2 \\ 2u & -3u & u-2 \end{vmatrix}$$

$$i\left[u^2(u-2) - (-3u(u+2))\right] - j\left[3u(u-2) - 2u(u+2)\right]$$
$$+ k\left[3u(-3u) - u^2(2u)\right]$$

$$i\left[u^3 - 2u^2 - (-3u^2 - 6u)\right] - j\left[3u^2 - 6u - 2u^2 - 4u\right]$$
$$+ k\left(-9u^2 - 2u^3\right)$$

$$i\left(u^3 - 2u^2 + 3u^2 + 6u\right) - j\left(3u^2 - 2u^2 - 6u - 4u\right)$$
$$+ k\left(-2u^3 - 9u^2\right)$$

$$i\left(u^3 + u^2 + 6u\right) - j\left(u^2 - 10u\right) + k\left(-2u^3 - 9u^2\right)$$

$$\bar{f} \times \bar{v} = \left(u^3 + u^2 + 6u\right)i - \left(u^2 - 10u\right)j + \left(2u^3 - 9u^2\right)k$$

$$\int (\bar{f} \times \bar{v})\, du = \int_0^1 \left(u^3 + u^2 + 6u\right)i - \left(u^2 - 10u\right)j + \left(2u^3 - 9u^2\right)k$$

$$= \int_0^1 (u^3+u^2+6u)\, i\, du - \int (u^2 - 10u)\, j\, du + \int (2u^3-9u^2)\, k\, du$$

$$\cancel{= \left|(u^3+u^2+6u)\right|_0^1 - \left|u^2 - 10u\right|_0 + \left|2u^3-9u^2\right|_0^1}$$

$$= \left|\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^2}{2}\right|_0^1 i - \left|\frac{u^3}{3} - \frac{10u^2}{2}\right|_0^1 j + \left|\frac{2u^4}{4} - \frac{9u^3}{3}\right|_0^1 k$$

$$= \left|\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right|_0^1 i - \left|\frac{u^3}{3} - 5u^2\right|_0^1 j + \left|\frac{u^4}{2} - 3u^3\right|_0^1 k$$

$$= \left[\left(\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2\right) - (0)\right] i - \left[\left(\frac{(1)^3}{3} - 5(1)^2\right) - (0)\right] j$$

$$+ \left[\left(\frac{(1)^4}{2} - 3(1)^3\right) - (0)\right] k$$

$$= \left(\frac{1}{4} + \frac{1}{3} + 3\right) i - \left(\frac{1}{3} - 5\right) j + \left(\frac{1}{2} - 3\right) k$$

$$= \left(\frac{43}{12}\right) i - \left(-\frac{14}{3}\right) j + \left(-\frac{5}{2}\right) k$$

$$= \frac{43}{12} i + \frac{14}{3} j - \frac{5}{2} k$$