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MATRIC NO: 19/ENG07/005

COURSE CODE: MAT 104

① $\int \frac{(3x-1)}{(x-1)(x-2)(x-3)}\,dx$

② $\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}\,dx$

③ $\int \frac{(x^2+1)}{(x-3)\cdot(x-2)^2}$

④ $\int \frac{(x^3+x^2+x+1)}{(x+1)}\,dx$

Solo

① $\frac{(3x-1)}{(x-1)(x-2)(x-3)}$

$\frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)} = \frac{3x-1}{(x-1)(x-2)(x-3)}$

$A(x-2)(x-3)+B((x-1)(x-3)+C(x-1)(x-2) = 3x-1$

$A(x^2-3x-2x+6)+B(x^2-3x-x+3)+C(x^2-2x-x+2) = 3x-1$

$A(x^2-5x+6)+B(x^2-4x+3)+C(x^2-3x+2) = 3x-1$

$Ax^2-5Ax+6A+Bx^2-4Bx+3B+Cx^2-3Cx+2C = 3x-1$

$(A+B+C)x^2 + (-5A-4B-3C)x + (6A+3B+2C) = 3x-1$

$A+B+C = 0$

$-5A-4B-3C = 3$

$6A+3B+2C = -1$

$A = -B-C$

$5(-B-C) - 4B - 3C = 3$

$-5B-5C-4B-3C = 3$

$-9B-8C = 3$ ——(1)

$6(-B-C) + 3B + 2C = -1$

$-6B-6C+3B+2C = -1$

$-3B-4C = -1$ ——(2)

By Elimination method

$-3 \times -9B-8C = 3$

$-9 \times -3B+4C = -1$

$\Rightarrow 27B+24C = -9$

$-27B+36C = 9$

$-12C = -18$

$-5A = 1$

By Elimination method

$-3 \times -9B-8C = 3$

$-9 \times -3B-4C = -1$

$\Rightarrow -27B+24C = -9$

$-27B+36C = 9$

$-12C = -18$

$C = \frac{-18}{-12}$

$C = \frac{3}{2}$

In eqn(2)

$-3B-4C = -1$

$-3B-4(\frac{3}{2}) = -1$

$-3B-6 = -1$

$-3B = 5$

$B = \frac{5}{-3}$

$B = -\frac{5}{3}$

$A + B - 1C = 0$

$A + \left(-\frac{5}{3}\right) + \frac{3}{2} = 0$

$A - \frac{1}{6} = 0$

$A = \frac{1}{6}$

$$\int \frac{\frac{1}{6}}{x-1}\,dx + \frac{\left(-\frac{5}{3}\right)}{x-2}\,dx + \frac{\frac{3}{2}\,dx}{x-3}$$

$$\frac{1}{6}\ln(x-1) - \frac{5}{3}\ln(x-2) + \frac{3}{2}\ln(x-3) + C$$

② $$\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}\,dx \qquad \int \frac{(x^2+x+1)}{(x^2+1)(x+2)}\,dx$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{Ax^2+Bx+C}{(x+2)} + \frac{D}{(x^2+1)} = \frac{(x^2+x+1)}{(x+2)(x^2+1)}$$

$$(Ax^2+Bx+C)(x^2+1) + D(x+2)$$

$$\frac{Ax^2+Bx+C}{(x^2+2)} + \frac{D}{(x+2)} = \frac{(x^2+x+1)}{(x^2+1)(x+2)}$$

$$\frac{(Ax^2+Bx+C)(x+2) + D(x^2+2)}{(x^2+2)(x+2)} = \frac{(x^2+x+1)}{(x^2+1)(x+2)}$$

Multiply B.S $(x^2+1)(x+2)$

$(Ax^2+Bx+C)(x+2)+D(x^2+1)=(x^2+x+1)$

$Ax^3+2Ax^2+Bx^2+2Bx+Cx+2C+Dx^2+D=(x^2+x+1)$

$(A)x^3+(2A+B+D)x^2+(2B+C)x+(2C+D)=(x^2+x+1)$

$A=0$ ——(1)

$2A+B+D=1$ ——(2)

$2B+C=1$ ——(3)

$2C+D=1$ ——(4)

Put $A=0$ in eqn(2)

$2(0)+B+D=1$

$B+D=1$

$B=-D$

Put $B=-D$ in (3)

$2(-D)+C=1$

$-2D+C=1$ ——(5)

$C-2D=1$ ——(6)

By elimination method

$1\times 2C+D=1$

$2\times C-2D=1$

$2C+D=1$

$-\ 2C-4D=2$

$-3D=-1$

$D=\frac{-1}{-3}$

$D=\frac{1}{3}$

$C-2D=1$

$C-2(\frac{1}{3})=1$

$C - \frac{2}{3} = 1$

$C = 1 - \frac{2}{3}$

$C = \frac{1}{3}$

$B = -D$

$B = -(\frac{1}{3})$

$B = -\frac{1}{3}$

$A = 0, B = -\frac{1}{3}, C = \frac{1}{3}, D = \frac{1}{3}$

$$\int \frac{0\,dx}{(x^2+1)} + \frac{(-\frac{1}{3})dx}{x^2+1} + \frac{(\frac{1}{3})dx}{x^2+1} + \frac{\frac{1}{3}dx}{x+2}$$

$$\Rightarrow -\frac{1}{3}\ln(x^2+1) + \frac{1}{3}\ln(x^2+1) - \frac{1}{3}\ln(x+2)$$

③ $\int \frac{(x^2+1)}{(x-3)(x-2)^2}\,dx$

$$\int \frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B(x-2)(x-3) + C(x-3)}{(x-3)(x-2)^2}$$

$$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$$

$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$

$x^2+1 = (A+B)x^2 + (-4A-5B+C)x + 4A - 6B - 3C$

$x^2+1 = (A+B = 1 \quad \text{---(1)}$

$-4A - 5B + C = 0 \quad \text{---(2)}$

$4A + 6B - 3C = 1 \quad \text{---(3)}$

$A = 1 - B \quad \text{---(4)}$

$-4(1-B) - 5B + C = 0$

$-4 + 4B - 5B + C = 0$

$-4 - B + C = 0$

$-B + C = +4$

$-B + C = 4$

$C = 4 + B$

$4(1-B) - 6B - 3C = 1$

$4 - 4B + 6B - 3C = 1$

$4 + 2B - 3C = 1$

$2B - 3C = -3$

$\times 10 \quad -4B + C = 4$

$-4 \quad -10B - 3C = -3$

$-10 \times -B + C = 4$

$-1 \quad -10B - 3C = -3$

$40B - 10C = -40$

$-40B + 12C = 12$

$+10B - 10C = -40$

$-10B + 3C = 3$

$-22C = -52$

$-13C =$

$-10B - 3(4 + B) = -3$

$-10B - 12 - 3B = -3$

$-10B - 3B = -3 + 12$

$-13B = 9$

$2x - B + C = 4$

$-1 \times 2B - 3C = -3$

$-2B + 2C = 8$

$-2B + 3C = 3$

$-C = 5$

$C = -5$

$-B + C = 4$

$-B - 5 = 4$

$-B = 4 + 5$

$-B = 9$

$B = -9$

$A + B = 1$

$A = 1 - B$

$A = 1 - (-9)$

$A = 10$

$$\int \frac{(x^2+1)}{(x-3)(x-2)^2}\,dx = \int \frac{10}{(x-3)}\,dx + \int \frac{-9\,dx}{x-2} + \int \frac{-5\,dx}{(x-2)^2}$$

$$= 10\ln(x-3) - 9\ln(x-2) - 5\ln(x-2)$$

$$= 10\ln(x-3) - 9\ln(x-2) - \frac{5}{x-2} + C$$

(4) $\int \frac{x^3+x^2+x+1}{x-1}\,dx$

$$\begin{array}{r|l} & x^2+2x+3 \\ x-1 & x^3+x^2+x+1 \\ & \underline{x^3-x^2} \\ & \quad 2x^2+x+1 \\ & \quad \underline{2x^2-2x} \\ & \quad\quad 3x+1 \\ & \quad\quad \underline{3x-3} \\ & \quad\quad\quad 4 \end{array}$$

which can now be written as

$$\int (x^3+2x+x+1)\,dx + \int \frac{4}{x-1}\,dx$$

$$\Rightarrow \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x$$

$$\Rightarrow \frac{x^3}{3} + x^2 + 3x + 4$$

$$\rightarrow \int (x^2+2x+3)\,dx + \int \frac{4}{x-1}\,dx$$

$$\rightarrow \frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + C$$